Solutions

~~Panther ID~~: _____ NAME(S): _____

MAC ~~2811~~ 2281 Worksheet ~~10~~ 2/05/2019 Group # =

1. Compute the derivatives of the following functions:

a) $\cot(x)$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x \cdot \frac{d}{dx}[\cos x] - \cos x \cdot \frac{d}{dx}[\sin x]}{(\sin x)^2}$$

$$= \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\overbrace{\sin^2 x + \cos^2 x}^{1})}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} = \boxed{-\csc^2 x}$$

b) $\sec(x)$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{\cos x \cdot \frac{d}{dx}[1] - 1 \cdot \frac{d}{dx}[\cos x]}{(\cos x)^2}$$

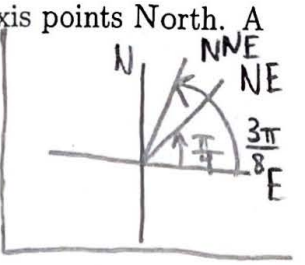
$$= \frac{\cos x \cdot 0 - (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \boxed{\tan x \sec x}$$

c) $\csc(x)$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{\sin x \cdot \frac{d}{dx}[1] - 1 \cdot \frac{d}{dx}[\sin x]}{(\sin x)^2}$$

$$= \frac{\sin x \cdot 0 - \cos x}{\sin^2 x} = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = \boxed{-\cot x \csc x}$$

2. Assume that the positive x-axis points East, and the positive y-axis points North. A train takes the path $y = x^2$ from $(0, 0)$ to $(1, 1)$.



a) Find all points where the train is pointing precisely Northeast.

Find where $y' = \tan\frac{\pi}{4} = 1$

$y' = 2x$ $\quad y = (\frac{1}{2})^2 = \frac{1}{4}$

$2x = 1$

$x = \frac{1}{2}$

The train points precisely Northeast at the point $(\frac{1}{2}, \frac{1}{4})$

b) Does the train ever point NNE?

Find where $y' = \tan(\frac{3\pi}{8}) \approx 2.41$

$y' = 2x$ $\quad y = (1.205)^2 \approx 1.452$

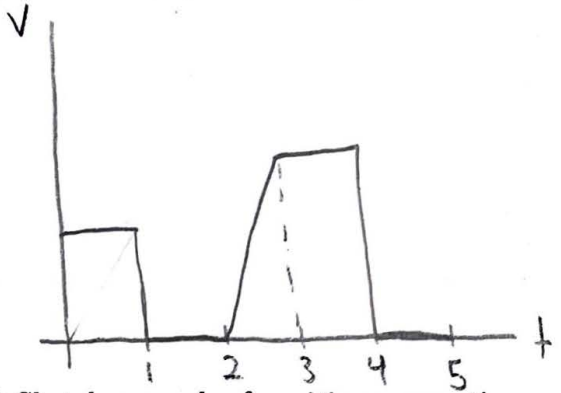
$2x \approx 2.41$

$x \approx 1.205$

The train would point NNE at the point $(1.205, 1.452)$. However, since this point does not fall on the path $y = x^2$ from $(0,0)$ to $(1,1)$, the train never points NNE.

3. Consider the following scenario. "A car is driving at 30 miles per hour, but then stops at a red light for 1 minute. At the red light, the driver agrees to race the car next to him. As soon as the light turns green, he accelerates to 100 miles per hour, maintains this speed for 1 minute, then crashes into a wall." Note: t is in minutes

a) Sketch a graph of velocity versus time.



b) Sketch a graph of position versus time.