Solutions

NAMES: ____________ ~~Panther ID:~~ ____________ NAMES: ____________

MAC 2311: Worksheet 2/12 Sp. 2019 (Chain Rule) Group # ____

1) Use the chain rule and other rules of differentiation as needed to compute the derivatives of the following functions

a) $\dfrac{1}{\sqrt{1+x^2}}$

$$\frac{d}{dx}\left[\frac{1}{\sqrt{1+x^2}}\right] = \left((1+x^2)^{-\frac{1}{2}}\right)' = -\frac{1}{2}(1+x^2)^{-\frac{3}{2}} \cdot (1+x^2)'$$

$$= -\frac{1}{\cancel{2}}(1+x^2)^{-\frac{3}{2}}(\cancel{2}x) = -x(1+x^2)^{-\frac{3}{2}} = \boxed{-\frac{x}{(1+x^2)^{\frac{3}{2}}}}$$

b) $\cos^2(3x)$

$$\frac{d}{dx}\left[\cos^2(3x)\right] = \left((\cos(3x))^2\right)' = 2\cos 3x \cdot (\cos 3x)'$$

$$= 2\cos 3x \cdot -\sin 3x \cdot (3x)' = 2\cos 3x \cdot -\sin 3x \cdot 3$$

$$= \boxed{-6\cos 3x \sin 3x}$$

*c) $x\left(x^9+2\right)^{1/2}$

$$\frac{d}{dx}\left[x(x^9+2)^{\frac{1}{2}}\right] = \left(x(x^9+2)^{\frac{1}{2}}\right)' = (x)'(x^2+9)^{\frac{1}{2}} + x\left((x^2+9)^{\frac{1}{2}}\right)'$$

$$= (x^2+9)^{\frac{1}{2}} + x \cdot \frac{1}{2}(x^2+9)^{-\frac{1}{2}} \cdot (x^2+9)' = (x^2+9)^{\frac{1}{2}} + x \cdot \frac{1}{\cancel{2}}(x^2+9)^{-\frac{1}{2}} \cdot \cancel{2}x$$

$$= \boxed{(x^2+9)^{\frac{1}{2}} + x^2(x^2+9)^{-\frac{1}{2}}} = (x^2+9)^{\frac{1}{2}} + \frac{x^2}{(x^2+9)^{\frac{1}{2}}} = \frac{(x^2+9)^{\frac{1}{2}}(x^2+9)^{\frac{1}{2}}}{(x^2+9)^{\frac{1}{2}}} + \frac{x^2}{(x^2+9)^{\frac{1}{2}}}$$

$$= \frac{x^2+9+x^2}{(x^2+9)^{\frac{1}{2}}} = \boxed{\frac{2x^2+9}{\sqrt{x^2+9}}}$$

* first answer is sufficient, second one is fully simplified

*d) $e^{x\cos(x)}$

$$\frac{d}{dx}\left[e^{x\cos x}\right] = \left(e^{x\cos x}\right)' = e^{x\cos x}\cdot(x\cos x)' = \boxed{e^{x\cos x}(\cos x - x\sin x)}$$

$$= \boxed{e^{x\cos x}\cos x - x e^{x\cos x}\sin x}$$

*e) $\sqrt{\csc(\sin^2 x)}$

$$\frac{d}{dx}\left[(\csc(\sin^2 x))^{\frac{1}{2}}\right] = \left((\csc(\sin^2 x))^{\frac{1}{2}}\right)' = \frac{1}{2}(\csc(\sin^2 x))^{-\frac{1}{2}}\cdot(\csc(\sin^2 x))'$$

$$= \frac{1}{2}(\csc(\sin^2 x))^{-\frac{1}{2}}\cdot -\csc(\sin^2 x)\cot(\sin^2 x)\cdot(\sin^2 x)'$$

$$= \frac{1}{2}(\csc(\sin^2 x))^{-\frac{1}{2}}\cdot -\csc(\sin^2 x)\cot(\sin^2 x)\cdot 2\sin x\cdot(\sin x)'$$

$$= \cancel{\frac{1}{2}}(\csc(\sin^2 x))^{-\frac{1}{2}}\cdot -\csc(\sin^2 x)\cot(\sin^2 x)\cdot\cancel{2}\sin x\cos x = \boxed{\frac{-\csc(\sin^2 x)\cot(\sin^2 x)\cos x\sin x}{(\csc(\sin^2 x))^{\frac{1}{2}}}}$$

$$= -(\csc(\sin^2 x))^{\frac{1}{2}}\cot(\sin^2 x)\cos x\sin x$$

$$= \boxed{-\sqrt{\csc(\sin^2 x)}\cot(\sin^2 x)\cos x\sin x}$$

f) $\sec(3 + x^2\tan(3x))$

$$\frac{d}{dx}\left[\sec(3+x^2\tan(3x))\right] = \left(\sec(3+x^2\tan(3x))\right)'$$

$$= \sec(3+x^2\tan(3x))\tan(3+x^2\tan(3x))\cdot(3+x^2\tan(3x))'$$

$$= \sec(3+x^2\tan(3x))\tan(3+x^2\tan(3x))\cdot(0 + 2x\tan 3x + x^2\sec^2(3x)\cdot(3x)')$$

$$= \sec(3+x^2\tan(3x))\tan(3+x^2\tan(3x))\cdot(2x\tan 3x + x^2\cdot\sec^2(3x)\cdot 3)$$

$$= \boxed{\sec(3+x^2\tan(3x))\tan(3+x^2\tan(3x))(2x\tan 3x + 3x^2\sec^2(3x))}$$

3) Consider the function $y = \sqrt{x^2 - 9}$. Find the equation of the line tangent to this function at $x = 5$.

$y = \sqrt{x^2 - 9}$

$y = \sqrt{(5)^2 - 9}$

$y = \sqrt{25 - 9}$

$y = \sqrt{16}$

$y = 4 \Rightarrow$ Point: $(5,4)$

$y' = \frac{1}{2}(x^2-9)^{-\frac{1}{2}}\cdot(x^2-9)'$

$y' = \cancel{\frac{1}{2}}(x^2-9)^{-\frac{1}{2}}\cdot(\cancel{2}x)$

$y' = x(x^2-9)^{-\frac{1}{2}}$

$y' = 5(5^2-9)^{-\frac{1}{2}}$

$y' = 5(25-9)^{-\frac{1}{2}}$

$y' = \frac{5}{\sqrt{16}} = \frac{5}{4} \Rightarrow$ Slope: $\frac{5}{4}$

Tangent Line

$\boxed{y - 4 = \frac{5}{4}(x-5)}$

$\Downarrow$

$y = \frac{5}{4}x - \frac{25}{4} + \frac{16}{4}$

$\boxed{y = \frac{5}{4}x - \frac{9}{4}}$