Name: Answer Key Panther ID: ____________

Exam 3 Calculus II Fall 2013

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (14 pts) In each case, answer True or False. No work or justification is needed. (2 pts each)

(a) Any bounded sequence is convergent. False

(b) Any convergent sequence is bounded. True

(c) If $\sum_{k=1}^{\infty} a_k$ converges, then $a_k \to 0$. True

(d) The series $1 - 1 + 1 - 1 + 1 - 1 + \ \dots$ converges to 0. False

(e) If $0 < k^2 a_k < 1$, for all $k \geq 1$, then $\displaystyle\sum_{k=1}^{\infty} a_k$ converges. True

(f) For any series with non-negative terms, if the series converges, then the series is absolutely convergent. True

(g) For any alternating series, if the series converges, then the series is absolutely convergent. False

2. (16 pts) The first five terms of a sequence are

$$a_1 = \frac{2}{3},\ a_2 = -\frac{4}{5},\ a_3 = \frac{6}{7},\ a_4 = -\frac{8}{9},\ a_5 = \frac{10}{11}, \dots$$

(a) (4 pts) Assuming that the sequence follows the indicated pattern, find the formula for the general term a_n.

$$a_n = (-1)^{n+1} \frac{2n}{2n+1}$$

(b) (4 pts) Is the sequence a_n convergent? Answer and briefly justify.

No, the subsequence $\{a_{2k+1}\}_k$ converges to 1, whereas the subsequence $\{a_{2k}\}_k$ converges to 1

(c) (4 pts) Is the sequence a_n bounded? Answer and briefly justify.

Yes $-1 \leq a_n \leq 1$ for all n.

(d) (4 pts) Is the sequence a_n eventually monotone? Answer and briefly justify.

No, the sequence is oscillating, so it is not eventually monotone

(E.g. $a_{2k-1} > a_{2k}$, but $a_{2k} < a_{2k+1}$ for all k)

These are just the answers and the tests used.
You solution should be more detailed by showing that you apply well the corresponding test.

3. (18 pts + bonus) Determine whether each of the following series converges or diverges. Be sure to state which test you are using and to show how it applies to the series in question (6 pts each, 2 pts answer, 4 pts justification).

Bonus: For each convergent series, if you can find the exact value of the sum, you will receive 3 bonus points.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{2k+1}{k^2+1}$ divergent series

comparison with harmonic series (limit or simple)

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+2k} = \frac{3}{4}$ convergent

telescopic

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{10^k}{k!} = e^{10} - 11$ convergent. Ratio Test

4. (18 pts) In each case, determine whether the series converges absolutely, converges conditionally, or diverges. Be sure to justify well your conclusions

(a) $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{2k-1}$

divergent, k^{th}-term test

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k\sqrt{k}}$ absolutely convergent

$\left(\displaystyle\sum_{k=2}^{\infty} \left|\frac{(-1)^k}{k\sqrt{k}}\right| = \sum_{k=2}^{\infty} \frac{1}{k^{\frac{3}{2}}}\right.$ conv.

p-series with $p = \frac{3}{2} > 1$

5. (14 pts) Find the interval of convergence of the series

$$\sum_{k=1}^{\infty} \frac{(x-3)^k}{k^2 \cdot 2^k}$$

. Be sure to also check the endpoints. $x \in [1, 5]$

7. (16 pts) (a) (6 pts) Find the MacLaurin polynomial of degree 5 for $f(x) = \sin x$.

(b) (4 pts) Use the MacLaurin polynomial in part (a) to get an approximation of sin 1 (here "1" means 1 radian).

(c) (6 pts) How many digits accuracy does your approximation in part (b) have?

8. (10pts) Choose ONE proof, explain thoroughly.

(a) State and prove the Simple Comparison Test for series.

(b) State and prove the formula for the sum of an infinite geometric series for the case $|r| < 1$.

See textbook or notes

7. (a) $P_5(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!}$

(b) $\sin(1) \approx \frac{1}{1!} - \frac{1}{3!} + \frac{1}{5!}$

(c) If you estimate the error with $n=5$, you'd get (M=1 since all derivatives of sin x are ± sin x, ± cos x, so in absolute value they are less than 1)

$|R_5(1)| \le \frac{1 \cdot |1-0|^{5+1}}{(5+1)!}$

so $|R_5(1)| \le \frac{1}{6!} = \frac{1}{720} < \frac{1}{100} = 10^{-2}$, so only 1 decimal accuracy

but since $P_5(x) = P_6(x)$ for sin x, in the error estimate we can actually use $n=6$ so

$|R_6(1)| \le$ [illegible] $= 10^{-3}$, so our approximation has [illegible]