

1. True or False. Answer and briefly justify in each case.

(a) If  $\{a_k\}_k$  is a convergent sequence then the series  $\sum_{k=1}^{\infty} a_k$  also converges.

(b) If the series  $\sum_{k=1}^{\infty} a_k$  converges, then the sequence  $\{a_k\}_k$  is convergent.

(c) If  $S_n = \sum_{k=1}^n a_k$  and  $\lim_{n \rightarrow \infty} S_n$  does not exist or is not finite, then  $\sum_{k=1}^{\infty} a_k$  is a divergent series .

(d) If  $\{a_k\}_k$  is a sequence convergent to 0 then the series  $\sum_{k=1}^{\infty} a_k$  converges.

(e) If  $\sum_{k=1}^{\infty} a_k$  converges to  $A$  and  $\sum_{k=1}^{\infty} b_k$  converges to  $B$ , then  $\sum_{k=1}^{\infty} (a_k - b_k)$  converges to  $A - B$  .

(f) If  $\sum_{k=1}^{\infty} a_k$  converges, then  $\sum_{k=1}^{\infty} (a_k)^2$  also converges.

2. (a) Use the ratio test to investigate if the series  $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$  converges or diverges .

(b) What can you say about  $\lim_{n \rightarrow \infty} \frac{n^n}{(2n)!}$  ?

3. Decide if each of the following series is absolutely convergent, conditionally convergent or divergent.

(a)  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

(b)  $\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$

(c)  $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$