Worksheet - 11/21/2013 MAC-2312 **Name:** ____________________

1. Use the definition, to find the Taylor series at $x_0 = 1$ for the function $f(x) = \ln x$. Find the interval of convergence of the series that you obtained. It can be shown that the series converges to $\ln x$ for all the values of x in the interval of convergence. Accepting this, what do you obtain when $x = 2$?

2. (a) Use the definition to find the MacLaurin series for $\sin x$ and $\cos x$.

(b) You need the value of $\cos(0.1)$ (where 0.1 is in radians) with an error of at most 10^{-10}, but you have no calculator. Write the Taylor polynomial of degree 6 for $\cos x$ and then use this polynomial to get an approximation of $\cos(0.1)$. Show that with this approximation the error is small enough. Use the remainder estimate formula

$$|R_n(x)| \leq \frac{M|x - x_0|^{n+1}}{(n+1)!} \text{ , where } M \text{ is an upper bound for the } (n+1)\text{-th derivative .}$$

(c) In the MacLaurin series of e^x, replace x by ix, where $i = \sqrt{-1}$. Using the pattern for powers of i, $i^0 = 1$, $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1 = i^0$, $i^5 = i = i^1$, etc, regroup in separate series the real and the imaginary terms from the series of e^{ix}. What do you observe? You should have discovered a famous formula of Euler:

$$e^{ix} = \cos x + i \sin x \ .$$

3. (a) Write the MacLaurin series for $f(x) = \frac{1}{1-x}$. What is the interval of convergence?

(b) Find the MacLaurin series for $g(x) = \frac{1}{1+x}$. Hint: Change $x \to -x$ in (a).

(c) Find the MacLaurin series for $h(x) = \frac{1}{1+x^2}$. Hint: Change $x \to x^2$ in (b).

(d) Find the MacLaurin series for $\arctan x$. Hint: Integrate (c).

Note: This is based on the following theorem (see Thm. 9.10.4 in the text): If a function is represented by a power series, then inside the interval of convergence (where the power series is absolutely convergent) one can differentiate or integrate term by term. The corresponding series will converge to the derivative or integral of the original function.