Name: _____ **Panther ID:** _____

Worksheet 5 Calculus II Fall 2013

1. (The Skydiver reloaded! – again adapted from Briggs) A skydiver in free fall subject to gravitational acceleration and air resistance has a velocity given by

$$v(t) = v_T\Big(\frac{e^{at}-1}{e^{at}+1}\Big)\ ,$$

where v_T is the terminal velocity and a is a physical constant. Find the distance the skydiver falls in the first t_0 seconds.

Hint: $e^{at} - 1 = 2e^{at} - (e^{at} + 1)$

2. The region bounded between the graph of $\sin x$ and the x-axis when $x \in [0, \pi]$ is rotated around the y-axis; the solid formed has volume V_1. Then the same region is rotated around the x-axis; the solid formed has volume V_2. Find V_1 and V_2 and observe that $V_1 = 4V_2$.

3. (a) Derive a reduction formula for

$$\int sin^n x\ dx\ ,$$

where n is a positive integer. You may check formula (9) in 7.2 to confirm your result.

(b) Use part (a) to derive a recursion formula for

$$A_n = \int_0^{\pi/2} sin^n x\ dx\ .$$

(c) Find A_1 (directly), A_3, A_5 (using the recursion formula), and then try to find a general formula for A_n when n is odd.

(d) Find A_0 (directly), A_2, A_4 (using the recursion formula), and then try to find a general formula for A_n when n is even.

The general formulas for A_n are the so-called *Wallis sine formulas.*