Worksheet 8 MAC-2312 - Fall '13 **Name:** _____

1. (a) An arithmetic sequence is a sequence $\{a_n\}$ defined recursively by $a_{n+1} = a_n + d$, for $n \geq 0$. Find the formula of the general term a_n of an arithmetic sequence in terms of the first term a_0 and the common difference d. Is an arithmetic sequence convergent or divergent? Justify. Is an arithmetic sequence monotone? Justify.

(b) A geometric sequence is a sequence $\{a_n\}$ defined recursively by $a_{n+1} = ra_n$, for $n \geq 0$. Find the formula of the general term a_n of an arithmetic sequence in terms of the first term a_0 and the common ratio r.

Depending on r, find $\lim_{n\to+\infty} a_n$ for a geometric sequence $\{a_n\}$ as above. *Hint*: Consider the cases $|r| < 1,\ r > 1,\ r < -1,\ r = 1,\ r = -1$, separately.

2. When left to drop to the floor, a certain rubber ball jumps back up to 2/3 of its original height. Suppose the process continues indefinitely. If the ball was dropped from a height of 6ft originally, what is the height that the ball reaches after the 3rd jump? After the nth jump? (*Hint:* Don't do the multiplications, try to find the pattern.)
Can you estimate the total distance traveled by the ball?

3. Consider the sequence:

$$a_1 = \sqrt{3},\ \ a_2 = \sqrt{3+2\sqrt{3}},\ \ a_3 = \sqrt{3+2\sqrt{3+2\sqrt{3}}},\ \ a_4 = \sqrt{3+2\sqrt{3+2\sqrt{3+2\sqrt{3}}}},\ \ ...$$

(a) Find a recursion formula for a_{n+1}.

(b) Use induction to prove that $0 \leq a_n \leq 3$, for all $n \geq 1$.

(c) Use induction to prove that the sequence $\{a_n\}$ is increasing.

(d) By (b) and (c) it follows that the sequence is convergent (why?). Find its limit.