**Name:** ______________________________ **Panther ID:** _______________

**Exam 3** Calculus II Fall 2014

**Important Rules:**

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.

2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.

3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.

4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

**1.** (12 pts) In each case answer True or False. No justification needed. (2 pts each)

(a) The curve $r = 1 - 2\sin\theta$ has a graph symmetrical with respect to the $y$-axis.

(b) If $\lim\limits_{k\to+\infty} a_k = 0$ then the series $\sum\limits_{k=1}^{\infty} a_k$ is convergent.

(c) If $0 < a_k < \frac{1}{k^2}$ for all $k \geq 1$, then $\sum\limits_{k=1}^{\infty} a_k$ is convergent.

(d) The alternating harmonic series is absolutely convergent.

(e) $1 + \dfrac{1}{2} - \dfrac{1}{2} + \dfrac{1}{2} - \dfrac{1}{2} + \dfrac{1}{2} - \dfrac{1}{2} + ... = 1$

(f) Suppose $a_k > 0$, $b_k > 0$, and $\lim\limits_{k\to+\infty} \dfrac{a_k}{b_k} = 0$. Then if $\sum\limits_{k=1}^{\infty} b_k$ converges, $\sum\limits_{k=1}^{\infty} a_k$ also converges .

**2.** (12 pts) (a) (6 pts) Sketch the graph of the limacon with inner loop $r = 1 - 2\sin\theta$. Be sure to give the polar coordinates of all points where the graph intersects the $x$-axis, the $y$-axis, or passes through the origin.

(b) (6 pts) Set up, but do **not** evaluate, an integral that represents the area inside the inner loop of the limacon $r = 1 - 2\sin\theta$.

**3.** (20 pts) Determine whether each of the following series converges or diverges. Full justification is required.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{2k+1}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

**4.** (20 pts) For each of the following series, determine if the series is divergent (D), conditionally convergent (CC), or absolutely convergent (AC). Answer **and carefully** justify your answer. Very little credit will be given just for a guess. Most credit is given for the quality of the justification. (10 pts each)

(a) $\displaystyle\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots$

(b) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{\pi^{2k}}{(2k)!}$

**5.** (14 pts) (a) (8 pts) Find the Taylor polynomial of degree 2 of $f(x) = \tan x$ at $x_0 = \pi/4$.

(b) (6 pts) Use this Taylor polynomial to get an approximation of $\tan(101\pi/400)$.

**6.** (a) (12 pts) Find the interval of convergence (with endpoints) of the power series $\displaystyle\sum_{k=1}^{\infty} \frac{k(x-2)^{k-1}}{3^k}$ .

(b) (6 pts) The series in part (a) is the Taylor series at $x_0 = 2$ of a certain function $f(x)$. Can you find the function $f(x)$? (*Hint:* Integrate, to first find $\int f(x)\, dx$.)

**7.** (12 pts) Choose ONE to prove:
(a) State and prove the $p$-series test (using the integral test).

(b) State and prove the area formula for polar coordinates. Be sure to have in your proof a picture, a sum, and a limit.

(c) Prove or disprove the statement in Pb. 1 (f): Suppose $a_k > 0$, $b_k > 0$, for all $k \geq 1$, and suppose $\lim_{k \to +\infty} \frac{a_k}{b_k} = 0$.

$$\text{Then if } \sum_{k=1}^{\infty} b_k \text{ converges, the series } \sum_{k=1}^{\infty} a_k \text{ also converges .}$$