Name: ______________________________ **Panther ID:** _______________

FINAL EXAM Calculus II Spring 2014

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.

2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.

3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.

4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (15 pts) Circle the correct answer. No justification is necessary for this problem.

(a) The arc length of $y = e^{2x}$ from $x = 2$ to $x = 4$ is given by

(i) $\dfrac{e^8 - e^4}{4-2}$ (ii) $\displaystyle\int_2^4 e^{2x}\,dx$ (iii) $\displaystyle\int_2^4 (1+e^{2x})\,dx$ (iv) $\displaystyle\int_2^4 \sqrt{1+4e^{4x}}\,dx$ (v) e^6

(b) The expression

$$\frac{d}{dx}\Big(\int_1^{x^3} \frac{\sin t}{t}\,dt\Big) \text{ is equivalent to}$$

(i) $\dfrac{\sin(x^3)}{x^3}$ (ii) $\sin 1$ (iii) $\dfrac{3\sin(x^3)}{x}$ (iv) $3x^2\dfrac{\sin t}{t}$ (v) 0

(c) Let $f(x)$ be an increasing continuous function on $[a, b]$ and let R_4 be the right end point Riemann sum approximation with 4 subdivisions of the integral $\int_a^b f(x)\,dx$. Then compared with the integral, R_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about f)

(d) The sequence $a_n = 2+(-1)^n,\ n \geq 1$ is

(i) convergent but not monotone (ii) monotone but divergent (iii) bounded but divergent (iv) eventually decreasing but unbounded (v) none of the above

(e) The average value of the function $f(x) = x^3$ over the interval $[0, 1]$ is

(i) $\dfrac{1}{2}$ (ii) $3x^2$ (iii) $\dfrac{1}{4}$ (iv) $\dfrac{1}{8}$ (v) none of these

2. (10 pts) Find the total distance traveled by a particle which moves along the x axis with a velocity $v(t) = 2t - 4$ meters/second, for $0 \leq t \leq 6$ seconds.

3. (14 pts) Find the volume of the solid that results when the region enclosed by $y = e^{-2x}$, $y = 0$, $x = 0$ and $x = 1$ is revolved about the x-axis. (Computation is required. Sketch of solid is also required.)

4. (10 pts) Evaluate the improper integral or show is divergent $\displaystyle\int_0^{+\infty} \frac{x}{x^2+1}\,dx$

5. (20 pts) Evaluate (10 pts each):

(a) $\displaystyle\int x \ln x \, dx$

(b) $\displaystyle\int \frac{1}{\sqrt{4+x^2}}\,dx$

6. (14 pts) Determine if the series $\sum_{k=1}^{\infty} \frac{2}{(2k-1)(2k+1)}$ converges. If so, find the sum of the series.

7. (12 pts) Determine if $\sum_{k=2}^{\infty} \frac{(-1)^k}{k(\ln k)^2}$ is absolutely convergent, conditionally convergent, or divergent. Justify.

8. (12 pts) Sketch the rose $r = \sin(2\theta)$ and compute the area of one petal.

9. (14 pts) Choose ONE:
(a) State and prove FTC part (b) (the one about $\frac{d}{dx}\int$).
(b) State and prove the the integration formula for area in polar coordinates. A picture, a sum and a limit should appear in your work.

10. (6 pts) Write the partial fraction decomposition. It is NOT required to determine the constants.

$$\frac{1}{(x+2)^3(x^2+4)^2} =$$

11. (14 pts) Find the radius and the interval of convergence for $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k (x-1)^k}{3^k \sqrt[3]{k}}$

12. (14 pts) (a) (6 pts) Write the Maclaurin series for e^x and use it to find a series whose sum is $1/\sqrt{e}$.

(b) (8 pts) What is the smallest n so that the partial sum S_n of the series in part (a) approximates $1/\sqrt{e}$ with an error less than 10^{-3}? Be sure to justify your answer.

13. (10 pts) Differentiate a familiar series to obtain the MacLaurin series for $\frac{1}{(1-x)^2}$.