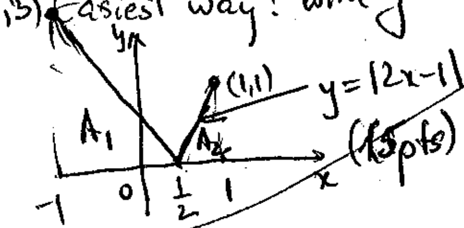
Quiz 1 MAC-2312 Fall 2014 **Name:** Answer & Grading Key

To receive credit you MUST SHOW ALL YOUR WORK.

1. Evaluate (3 pts each):

(a) $\int_{-1}^{1} |2x-1|\, dx = *$

(1.5) Easiest way: with geometry



$* = A_1 + A_2 =$

$= \frac{3 \cdot \frac{3}{2}}{2} + \frac{1 \cdot \frac{1}{2}}{2} = \frac{9}{4} + \frac{1}{4}$

$= \frac{10}{4} = \boxed{\frac{5}{2}}$ (1.5pts)

(b) Area under $g(x) = \frac{1}{3x+1}$ when $x \in [0,1]$

$A \underset{(0.5pts)}{=} \int_0^1 \frac{1}{3x+1}\, dx \overset{(2pts)}{=} \frac{1}{3} \ln(3x+1) \Big|_{x=0}^{x=1}$

Guess & Adjust!

$= \frac{1}{3}\ln 4 - \frac{1}{3}\ln 1 = \boxed{\frac{1}{3}\ln 4}$ (0.5pts)

(0.5pts)

$A = \int_0^1 \frac{1}{3x+1} dx$ with substitution

Let $u = 3x+1$
$du = 3dx$
$\frac{1}{3}du = dx$ (1pt)

$\underset{(0.5pts)}{=} \int_{u=1}^{u=4} \frac{1}{u} \frac{1}{3} du \overset{(0.5pts)}{=} \frac{1}{3} \ln u \Big|_{u=1}^{u=4}$

$= \ldots = \boxed{\frac{1}{3}\ln 4}$

(0.5pts)

(c) $\int_0^{\sqrt{3}} x\sqrt{1+x^2}\, dx =$

substitution
$w = 1+x^2$
$dw = 2x\,dx$
$\frac{1}{2}dw = x \cdot dx$ (1pt)

(1pt)

$= \int_{w=1}^{w=4} \sqrt{w} \cdot \frac{1}{2} dw = \frac{1}{2} \int_1^4 w^{\frac{1}{2}} dw$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} w^{\frac{3}{2}} \Big|_{w=1}^{w=4} =$

$= \frac{1}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{1}{3}(8-1) = \boxed{\frac{7}{3}}$

(1pt)

(d) Average value of $f(x) = \frac{1}{\sqrt{1-x^2}}$ when $x \in [0, \frac{1}{2}]$

(1pt)

$f_{ave} \overset{(1pt)}{=} \frac{\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx}{\frac{1}{2} - 0} = \frac{\arcsin(x) \Big|_{x=0}^{x=\frac{1}{2}}}{\frac{1}{2}}$

$f_{ave} = 2\left(\arcsin\left(\frac{1}{2}\right) - \arcsin(0)\right)$

$f_{ave} = 2 \cdot \left(\frac{\pi}{6} - 0\right) = \boxed{\frac{\pi}{3}}$ ← (1pt)