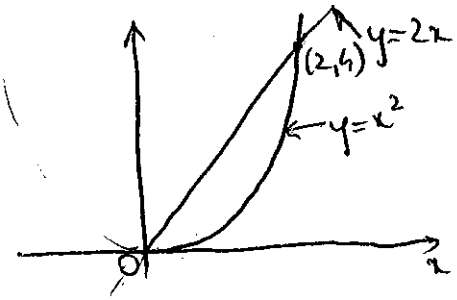
Quiz 2 - MAC 2312 Fall 2014 NAME: Solution Key

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (a) (5 pts) Find the area of the region between the curves $y = x^2$, $y = 2x$. Sketch of the region and computation are required.



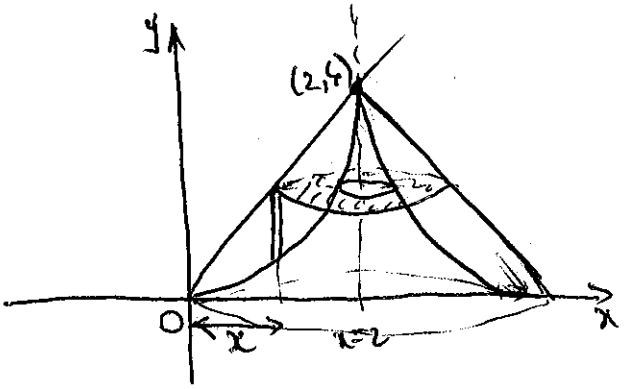
Intersection pts

$\begin{cases} y = x^2 \\ y = 2x \end{cases} \Rightarrow x^2 = 2x \Rightarrow x = 0, x = 2$

$$A = \int_{x=0}^{x=2} (2x - x^2)\,dx = \left(x^2 - \frac{x^3}{3}\right)\Big|_0^2$$

$$A = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$$

(b) (5 pts) The region in part (a) is rotated around the vertical line $x = 2$. Sketch the solid and then set up, but *do not evaluate*, an integral which represents the volume of this solid. You'll receive 2 bonus points if you give two correct solutions for part (b), one using slices and the other using cylindrical shells.



Sol.1 : (Slices)

$$V = \int_?^? A_{slice} \cdot Th_{slice}$$

$Th_{slice} = dy$

$A_{slice} = \pi(R^2 - r^2)$

$R = 2 - x_{line} = 2 - \frac{y}{2}$ $\quad r = 2 - x_{parab.} = 2 - \sqrt{y}$

$$\boxed{V = \pi \int_{y=0}^{y=4} \left[\left(2 - \frac{y}{2}\right)^2 - \left(2 - \sqrt{y}\right)^2\right] dy}$$

Sol.2 (Cyl. Shells)

$$V = \int_?^? 2\pi R_{shell} \cdot h_{shell} \cdot Th_{shell}$$

$Th_{shell} = dx$

$h_{shell} = y_{line} - y_{parab.} = 2x - x^2$

$R_{shell} = 2 - x$

$$\boxed{V = 2\pi \int_{x=0}^{x=2} (2 - x)(2x - x^2)\,dx}$$