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Worksheet week 2 - MAC 2312, Fall 2014

1. (Problem 62, section 5.4 textbook)

(a) Find a simple closed form for the sum $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+...+\frac{1}{n(n+1)}$

Hint: Check that $\frac{1}{k(k+1)}=\frac{1}{k}-\frac{1}{k+1}$ and, using this, observe that you get a telescopic sum .

(b) Use the result in part (a) to find

$$\lim_{n\to+\infty}\sum_{k=1}^{n}\frac{1}{k(k+1)}$$

Note: This limit is the definition of the *infinite series* $\sum_{k=1}^{+\infty}\frac{1}{k(k+1)}$.

2. (a) For any $r\neq 1$, prove the identity $1+r+r^2+r^3+...+r^n=\frac{1-r^{n+1}}{1-r}$

(b) Use part (a) to find the values of r for which the limit below exists and is finite and compute the limit

$$\lim_{n\to+\infty}\sum_{k=0}^{n}r^k$$

Note: This limit represents the *geometric series* $\sum_{k=0}^{+\infty}r^k$.

(c) Use part (b) to compute the infinite sum $1/2+1/4+1/8+1/16+...$

(d) Use part (b) to express as a fraction the number 0.377777...

3. (Pb. 53 section 5.4 textbook) (a) Use the right-end point Riemann sum to show that the area under the graph of $f(x)=x^3$ and over the interval $[0,b]$ is $b^4/4$.

(b) Use part (a) to find the area under the graph of $f(x)=x^3$ and over the interval $[a,b]$, where $0\leq a\leq b$.