Name: _____ **Panther ID:** _____

Exam 1 Calculus II Fall 2012

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (15 pts) For (a) and (b) circle the correct answer; for (c) write the answer.

(a) A particle is moving along the x-axis and its position at time t is given by the coordinate at that moment $x(t)$. The total distance traveled by the particle in the time interval $[t_1, t_2]$ is given by

(i) $\dfrac{x(t_2) - x(t_1)}{t_2 - t_1}$ (ii) $x(t_2) - x(t_1)$ (iii) $\displaystyle\int_{t_1}^{t_2} x(t)\,dt$ (iv) $\displaystyle\int_{t_1}^{t_2} |x'(t)|\,dt$ (v) $\displaystyle\int_{t_1}^{t_2} x'(t)\,dt$

(b) The average value of $f(x) = x^2$ on the interval $[0, 4]$ is

(i) 0 (ii) 4 (iii) 8 (iv) $\dfrac{16}{3}$ (v) 2

(c) Suppose sludge is emptied into a river at a rate of $r(t)$ gallons per minute, starting at time $t = 0$. Write an integral to represent the total amount of sludge that is emptied into the river during the first hour.

2. (15 pts) Let $F(x) = \int_0^x \sqrt{9 - t^2}\,dt$. Find each of the following:

(a) (2 pts) $F(0)$ (b) (4 pts) $F(3)$ (c) (4 pts) $F'(0)$ (d) (5 pts) $F''(0)$

3. (20 pts) Compute each integral (10 pts each):

(a) $\displaystyle\int_{-1}^{0} 6t^2(t^3+1)^{19}\,dt$

(b) $\displaystyle\int_{0}^{\ln(\sqrt{3})} \frac{e^x}{1+e^{2x}}\,dx$

4. (20 pts) Set up integrals to represent each of the following (you **do not** have to evaluate).

(a) The area of the region enclosed by the curves $y=\sqrt{x}$, $y=2-x$, $y=0$. A sketch of the region is required for full credit.

(b) The volume of the solid generated when the region in the first quadrant enclosed between $y=x$ and $y=x^2$ is revolved around the line $x=1$. Sketch of solid is required.

5. (10 pts) Compute the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+...+\sqrt{n}}{n\sqrt{n}} ,$$

by recognizing the sum as the Riemann sum of a certain function on the interval $[0, 1]$ and then computing the corresponding integral.

6. (15 pts) Newton's Law of gravitation states that the gravitational force exerted by the Earth on an object is proportional to the inverse of the square distance between the object and the center of the Earth. Thus, an object's weight $w(x)$ is related to its distance x from the Earth's center by the formula

$$w(x) = \frac{k}{x^2} , \quad \text{where } k \text{ is a constant that depends on the mass of the object.}$$

A satellite has a weight of 1000 lbs on the surface at the Earth (that is, when $x = 4000$ miles).

(a) Show that the constant k for this satellite is $k = 16 \times 10^9$.

(b) Set up an integral (you **do not** have to evaluate) that represent the total work required to lift the satellite from the surface of the Earth to an orbital position that is 1000 miles high.

7. (15 pts) Show that the surface area of a spherical cap of height h cut from a sphere of radius r is given by $S = 2\pi rh$.
Hint: Revolve an appropriate portion of the circle $x(t) = r\cos t\ ,\quad y(t) = r\sin t$ around the y-axis. Set up an integral for the surface area and then evaluate this integral.