Name: ______________________________ **Panther ID:** _______________

Exam 3 Calculus II Fall 2012

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (20 pts) The first five terms of a sequence are

$$a_1 = \frac{1}{2},\ a_2 = \frac{3}{4},\ a_3 = \frac{5}{6},\ a_4 = \frac{7}{8},\ a_5 = \frac{9}{10},\ldots$$

(a) (5 pts) Assuming that the sequence follows the indicated pattern, find the formula for the general term a_k.

(b) (5 pts) Using your answer from (a), is the sequence $\{a_k\}$ monotone? Justify your answer.

(c) (5 pts) Is the sequence $\{a_k\}$ convergent or divergent? Briefly justify your answer.

(d) (5 pts) Is the series $\sum_{k=1}^{\infty} a_k$ convergent or divergent? Briefly justify your answer.

2. (10 pts) A rubber ball is dropped from a height of 50 m. Each time it strikes the ground it bounces vertically to a height that is $\frac{2}{3}$ of the preceding height. Find the total distance the ball will travel if it is assumed to bounce infinitely often.

3. (16 pts) Find the exact value of the sum for each of the following series. Justification of the answer is required.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{4k^2 - 1}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{10^k}{k!}$

4. (14 pts) (a) (6 pts) Write in summation notation the Taylor series for a function $f(x)$ at a point x_0. Write also in expanded form each of the Taylor polynomials of degree $n = 1$, $n = 2$, $n = 3$ at x_0. The Taylor polynomial of degree 1 is said to be a linear approximation of the function, the one of degree 2 is said to be a quadratic approximation of the function, the one of degree 3 a cubic approximation, etc.

(b) (8 pts) Find the Taylor polynomial of degree 2 for the function $f(x) = \sqrt[3]{x}$ at $x_0 = 1$. Use this to obtain an approximation for $\sqrt[3]{1.2}$.

5. (24 pts) Determine whether each of the following series converges or diverges. Full justification is required.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{k^2+1}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k-1)!}$

6. (a) (12 pts) Find the interval of convergence (with endpoints) of the series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}$.

(b) (8 pts) The series in part (a) is the MacLaurin series expansion of a familiar function $f(x)$. Find $f(x)$. (*Hint:* First find $f'(x)$ by differentiating the series.)

7. (12 pts) Determine whether the series

$\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\ldots$ converges absolutely, converges conditionally, or diverges.

If you see the link with Problem 6, you may be able to say more about this series and you'd be rewarded with 4 more bonus points!