Worksheet - 03/19/13 MAC-2312 **Name:** ____________________________

1. (a) An arithmetic sequence is a sequence $\{a_n\}$ defined recursively by $a_{n+1} = a_n + d$, for $n \geq 0$. Find the formula of the general term a_n of an arithmetic sequence in terms of the first term a_0 and the common difference d. Is an arithmetic sequence convergent or divergent? Justify. Is an arithmetic sequence monotone? Justify.

(b) A geometric sequence is a sequence $\{a_n\}$ defined recursively by $a_{n+1} = ra_n$, for $n \geq 0$. Find the formula of the general term a_n of an arithmetic sequence in terms of the first term a_0 and the common ratio r.

Depending on r, find $\lim_{n\to+\infty} a_n$ for a geometric sequence $\{a_n\}$ as above. *Hint*: Consider the cases $|r| < 1$, $r > 1$, $r < -1$, $r = 1$, $r = -1$, separately.

2. Consider the sequence:

$$a_1 = \sqrt{3},\ \ a_2 = \sqrt{3 + 2\sqrt{3}},\ \ a_3 = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3}}},\ \ a_4 = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + 2\sqrt{3}}}},\ \ldots$$

(a) Find a recursion formula for a_{n+1}.

(b) Use induction to prove that $0 \leq a_n \leq 3$, for all $n \geq 1$.

(c) Use induction to prove that the sequence $\{a_n\}$ is increasing.

(d) By (b) and (c) it follows that the sequence is convergent (why?). Find its limit.

3. A trust fund is designed to pay to you and your descendants one thousand dollars per year in perpetuity. If inflation depreciates those dollars at 5 percent per year, what is the value of this fund in the long run in terms of the current money?