Name: ____________________ **Panther ID:** __________

Exam 2 Calculus II Spring 2019

To receive credit you MUST SHOW ALL YOUR WORK.

1. (12 pts) Circle the correct answer (3 pts each):

(a) For the integral $\int \sqrt{9x^2+4}\,dx$, the following substitution is helpful:

(i) $x = \tan\theta$ (ii) $3x = 2\sin\theta$ (iii) $x = 3\sec\theta$ (iv) $3x = 2\tan\theta$ (v) $w = 9x^2 + 4$

(Don't spend time evaluating the integral. It is not required.)

(b) The partial fraction decomposition for $\frac{x+3}{(x+2)^2(x^2+4)}$ is of the form:

(i) $\dfrac{A}{x+2} + \dfrac{B}{x^2+4}$ (ii) $\dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{Cx+D}{x^2+4}$ (iii) $\dfrac{x+3}{(x+2)^2} + \dfrac{x+3}{x^2+4}$

(iv) $\dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{(x+2)^3} + \dfrac{D}{(x+2)^4}$ (v) none of the above

(c) A function $f(x)$ is known to be continuous, positive and concave down when $x \in [-2,2]$. Let T_4 be the trapezoid approximation with 4 subdivisions of the integral $\int_{-2}^{2} f(x)\,dx$. Then compared with the integral, T_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about f)

(d) A function $f(x)$ is known to be continuous, positive and concave down when $x \in [-2,2]$. Let R_4^{right} be the right end-point approximation with 4 subdivisions of the integral $\int_{-2}^{2} f(x)\,dx$. Then compared with the integral, R_4^{right} is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about f)

2. (8 pts) Write an expression corresponding to M_4, the midpoint approximation with 4 subdivisions, for the integral $\int_0^1 e^{-t^2}\,dt$. Leave your answer in a calculator ready form, but you do not need to try to evaluate.

For Problems 3-6, evaluate each integral.

3. (10 pts) $\displaystyle\int_0^1 \frac{x}{4x^2+1}\,dx$

4. (10 pts) $\displaystyle\int x^2 e^{2x}\,dx$

5. (10 pts) $\displaystyle\int \tan^3 x \sec x \, dx$

6. (12 pts) $\displaystyle\int_0^2 \frac{x^3}{\sqrt{4-x^2}} \, dx$

7. (14 pts) Use partial fractions (or any other method) to compute

$$\int \frac{x+2}{x(x^2+4)}\, dx$$

8. (10 pts) The graph of $y = x^2$ for $0 \leq x \leq 2$ is revolved about the y-axis to form a tank that is then filled with salt water from the Dead Sea (weighing approximately 73 lb/ft^3). How much work does it take to pump all of the water to the top of the tank? (Assume both x and y are measured in feet.) Just set-up of the integral is required, you **DO NOT** have to evaluate the integral.

9. (10 pts) The following reduction formula holds for any positive constant a and any integers $m \geq 1$ and $n \neq -1$.

$$\int x^n (\ln ax)^m \, dx = \frac{x^{n+1}(\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} \, dx$$

Use the above reduction formula to compute

$$\int x^3 (\ln 2x)^2 \, dx$$

10. (10 pts) Choose ONE proof. If you do both, only the larger score will be considered for this problem, but the second score may give some bonus towards a previous problem where your score is smaller.

(A) State and prove the integration by parts formula.

(B) Prove the reduction formula stated in Problem 2:

$$\int x^n (\ln ax)^m \, dx = \frac{x^{n+1}(\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} \, dx$$