Name: ____________________ **Panther ID:** __________

Exam 3 Calculus II Spring 2019

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (12 pts) The first six terms of a sequence $\{a_n\}_n$ are given below

$$a_1 = \frac{1}{3},\ a_2 = -\frac{2}{5},\ a_3 = \frac{3}{7},\ a_4 = -\frac{4}{9},\ a_5 = \frac{5}{11},\ a_6 = -\frac{6}{13},\ ...$$

(a) (6 pts) Assuming that the pattern continues, find the formula for the general term a_n.

(b) (6 pts) Using your answer from (a), is the sequence $\{a_n\}_n$ convergent? Briefly justify your answer.

2. (12 pts) Evaluate each of the following (or show it diverges) (6 pts each)

(a) $\displaystyle\int_0^\infty \frac{1}{1+x^2}\, dx$

(b) $\ln(1/3) + \ln(3/5) + \ln(5/7) + \ln(7/9) + ...$

3. (12 pts) Evaluate each of the following (or show it diverges) (6 pts each)

$(a)\ \lim_{k \to +\infty} \left(\frac{3}{2^k} - \frac{2}{3^k} \right)$

$(b)\ \sum_{k=0}^{+\infty} \left(\frac{3}{2^k} - \frac{2}{3^k} \right)$

4. (12 pts) (a) (6 pts) Sketch the graph of the limacon with inner loop $r = 1 + 2\cos\theta$. Be sure to give the polar coordinates of all points where the graph intersects the x-axis, the y-axis, or passes through the origin.

(b) (6 pts) Set up an integral that represents the area inside the inner loop of the limacon $r = 1 + 2\cos\theta$. Just set up of the integral is required, you **do NOT** have to compute the integral.

5. (10 pts) In each case, answer True or False. No justification is required for this problem. (2 pts each)

(a) The graph of $r = 4\cos\theta$ in the Cartesian xy-plane is a circle that goes through the origin. **True** **False**

(b) If $a_k \geq \frac{1}{\sqrt{k}}$ for all $k \geq 1$, then $\sum_{k=1}^{\infty} a_k$ is divergent. **True** **False**

(c) If $\lim_{k\to+\infty} a_k = 0$ then $\sum_{k=1}^{\infty} a_k$ is convergent. **True** **False**

(d) If $\lim_{k\to+\infty} \sqrt[k]{|a_k|} = \frac{4}{5}$ then $\sum_{k=1}^{\infty} a_k$ is absolutely convergent. **True** **False**

(e) Any bounded sequence is convergent. **True** **False**

6. (16 pts + 4 bonus pts) Determine whether each of the following series converges or diverges. Full justification is required. If you determine the precise value of the sum, you'll receive up to 4 bonus points.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{2k+1}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{4k^2-1}$

7. (20 pts) For each of the following series, determine if the series is absolutely convergent (AC), conditionally convergent (CC), or divergent (D). Answer **and carefully** justify your answer. Very little credit will be given just for a guess. Most credit is given for the quality of the justification. (10 pts each)

(a) $\displaystyle\sum_{k=2}^{\infty}(-1)^k\frac{1}{k\ln k}$

(b) $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{(k!)^2}{(2k+1)!}$

8. (12 pts) Choose ONE to prove:

(a) State and prove the geometric series theorem.

(b) State and prove the p-series test, using the integral test.