Homework: Due Tuesday, Feb. 19 - MAC 2312, Spring 2019 NAME: Solution Key

1. (5 pts) Use integration by parts to find

$$\int \arcsin x \, dx =$$

I.B.P. $du = dx$ $v = \arcsin x$

$u = x$ $dv = \frac{1}{\sqrt{1-x^2}}\,dx$

$$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx = \boxed{x\arcsin x + \sqrt{1-x^2} + c}$$

sub. $w = 1-x^2$ | so $\int \frac{x}{\sqrt{1-x^2}}\,dx = \int \frac{-\frac{1}{2}dw}{\sqrt{w}}$

$dw = -2x\,dx$

$-\frac{1}{2}dw = x\,dx$ | $= -\frac{1}{2}\int w^{-\frac{1}{2}}dw$

$= -\frac{1}{\cancel{2}}\cancel{2}w^{\frac{1}{2}} + c = -\sqrt{1-x^2}$

2. (6 pts) Obtain the reduction formula

$$\int (\ln x)^n \, dx = x(\ln x)^n - n\int (\ln x)^{n-1}\, dx \qquad (*)$$

and use it to compute the exact value of $\int_1^e (\ln x)^3 \, dx$.

Use I.B.P. for $\int (\ln x)^n dx$ with $du = dx$ and $v = (\ln x)^n$

Then $u = x$ and $dv = n(\ln x)^{n-1}\cdot\frac{1}{x}dx$

$$\int(\ln x)^n dx = x(\ln x)^n - n\int \cancel{x}\cdot\frac{1}{\cancel{x}}(\ln x)^{n-1}dx,$$

so we got the reduction formula above.

Using it for $n=3$

$$\int(\ln x)^3 dx = x(\ln x)^3 - 3\int(\ln x)^2 dx \overset{\text{again } (*) \text{ for } n=2 \text{ next}}{=}$$

$$= x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int(\ln x)^1 dx\right] \overset{(*) \text{ for } n=1}{=}$$

$$= x(\ln x)^3 - 3x(\ln x)^2 + 6\left[x\ln x - \int 1\,dx\right]$$

Thus $\int(\ln x)^3 dx = x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + c$

$$\int_1^e (\ln x)^3 dx = e(\ln e)^3 - 3e(\ln e)^2 + \cancel{6e\ln e} - \cancel{6e} - \left(1\cancel{(\ln 1)^3} - 3\cdot 1\cancel{(\ln 1)^2} + \cancel{6\cdot\ln 1} - 6\right)$$

$$= \boxed{6-2e}$$