Quiz 3 MAC-2312 Spring 2019 **Name:** Solution Key

To receive credit you MUST SHOW ALL YOUR WORK.

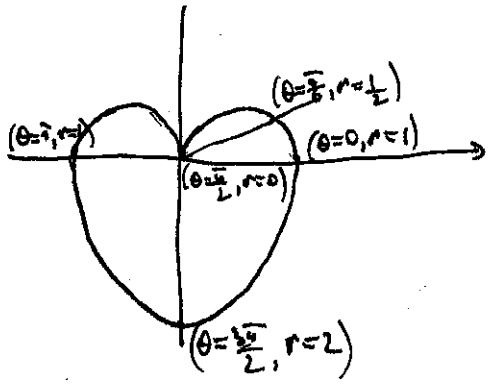
1. (3 pts) Replace the polar equation $r\cos\theta + r\sin\theta = 1$ with an equivalent Cartesian equation. Then describe or identify the graph.

As $x = r\cos\theta$ and $y = r\sin\theta$, the equation above is equivalent to $x + y = 1$. This is a line.

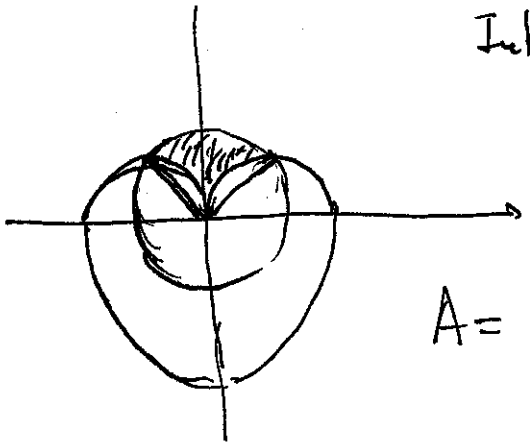
2. (a) (4 pts) Sketch the cardioid $r = 1 - \sin\theta$ in the Cartesian xy-plane. Be sure to indicate the axis of symmetry and give the polar coordinates of at least 5 points.

Symmetrical w.r.t. y-axis (as $\sin(\frac{\pi}{2} - \theta) = \sin\theta$)

θ	r
0	1
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0
$\frac{5\pi}{6}$	$\frac{1}{2}$
$\frac{3\pi}{2}$	2



(b) (4 pts) Set up an integral (or sum/difference of integrals) that represents the area of the region inside the circle $r = 1/2$, but outside the cardioid $r = 1 - \sin\theta$. You DO NOT have to spend time evaluating the integral. It is not required for this quiz.



Intersection of the two curves

$$\begin{cases} r = \frac{1}{2} \\ r = 1 - \sin\theta \end{cases} \Rightarrow \frac{1}{2} = 1 - \sin\theta$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta_1 = \frac{\pi}{6},\ \theta_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$A = \frac{1}{2}\int_{\theta=\frac{\pi}{6}}^{\theta=\frac{5\pi}{6}} \left(\left(\frac{1}{2}\right)^2 - (1-\sin\theta)^2\right) d\theta$$