Group #

Worksheet week 2 - MAC 2312, Spring 2019 NAME: Solution key

1. Use FTC or geometry to evaluate each integral:

(a) $\int_1^2 \frac{x^2+1}{x}\,dx = \int_1^2 \left(x+\frac{1}{x}\right)dx$

$= \left(\frac{x^2}{2} + \ln|x|\right)\Big|_{x=1}^{x=2}$

$= \left(\frac{2^2}{2} + \ln 2\right) - \left(\frac{1^2}{2} + \ln 1\right)$

$= \boxed{\frac{3}{2} + \ln 2}$

(b) $\int_0^{\pi/3} \sec^2 x\,dx = \left(\tan x\right)\Big|_{x=0}^{x=\frac{\pi}{3}}$

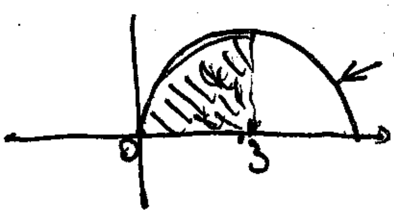
$= \tan\frac{\pi}{3} - \tan 0 = \sqrt{3} - 0$

$= \boxed{\sqrt{3}}$

(c) $\int_0^3 \sqrt{6x-x^2}\,dx$ Hint: Complete the square and graph $y=\sqrt{6x-x^2}$

$6x - x^2 = -(x^2-6x) = -(x^2-6x+9-9) = -((x-3)^2-9) = 9-(x-3)^2$

Thus $y = \sqrt{6x-x^2} = \sqrt{9-(x-3)^2}$, so its graph is a semi-circle centered at $(3,0)$ of radius 3.



$y=\sqrt{9-(x-3)^2}$

The integral $\int_0^3 \sqrt{6x-x^2}\,dx = \int_0^3 \sqrt{9-(x-3)^2}\,dx$ represents a quarter of the area of a disk with radius 3, so $\int_0^3 \sqrt{6x-x^2}\,dx = \frac{\pi\cdot 3^2}{4} = \boxed{\frac{9\pi}{4}}$

2. Suppose a gauge at the outflow of a reservoir measures the flow rate of water $r(t)$, in ft^3/min, at t minutes since the valve is open.

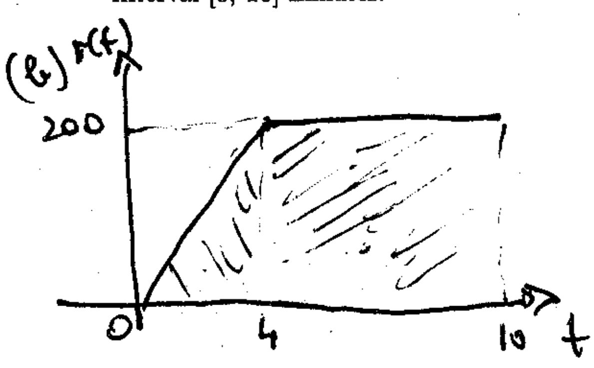
(a) In one sentence, explain what the following integral represents: $\int_2^6 r(t)\,dt$

The total amount of water that flowed from the tank in the interval $[2,6]$ min

(b) Suppose now the flow rate is given by the function $r(t) = \begin{cases} 50t & \text{if } 0 \le t \le 4 \\ 200 & \text{if } 4 < t \le 10 \end{cases}$ Graph this function.

(c) With the function $r(t)$ from part (b), find the total amount of water that flows out of the reservoir in the interval [0, 10] minutes.

(b)



$\int_0^{10} r(t)\,dt = \int_0^4 50t\,dt + \int_4^{10} 200\,dt$

$= 400 + 1200 =$

either geometry or FTC

$= \boxed{1600\ \text{ft}^3}$