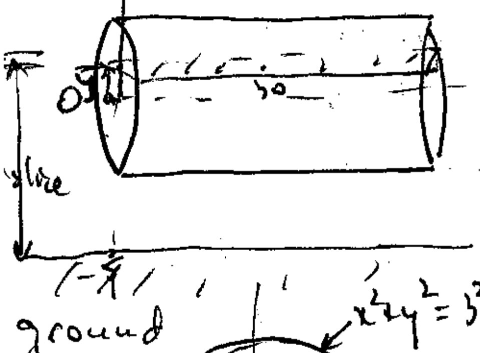
Solution Key for Worksheet 2/08

See your notes for the solution with the origin chosen at ground level!

1. The tank of a fuel truck is a cylinder of radius 3 ft and length 30 ft. The tank sits horizontally with the lower side at an altitude of 2 ft above the ground (wheels of the truck are 2 ft high). Assuming that the tank is initially half-full, set up an integral that represents the total work required to completely fill up the tank by pumping up gasoline from ground level. The density of gasoline is $\rho = 45$ lb/ft^3. (Just set up. The calculation of the integral is not required. Make sure to show on a picture what variable(s) you are using.)

Also Solved in class ~~see your notes~~ but with a different choice of origin (I think)

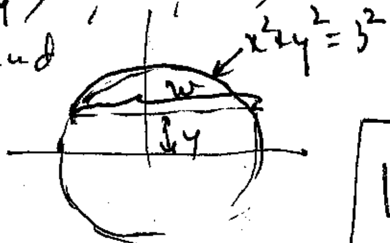


$$W = \int_?^? dW = \int_?^? \rho \cdot A_{slice} \cdot d_{slice} \cdot Th_{slice}$$

Will choose the origin at the center of the circle of the left side of the tank

$Th_{slice} = dy \quad d_{slice} = y-(-5) = \underline{\underline{y+5}}$

$A_{slice} = 30 \cdot \underset{\text{width}}{w} = 30 \cdot 2x = 60\sqrt{9-y^2}$



$$\boxed{W = \int_{y=0}^{y=3} 45 \cdot 60\sqrt{9-y^2} \cdot (y+5)\, dy}$$

$y=0$ ← tank is initially half-full

2. Suppose you have to drill a narrow but deep pit into the ground. The pit is cylindrical, with a radius of 1ft and with a depth of 1000ft. The density of the rock encountered varies, so assume that at a depth of x ft from the ground, the density is given by some function $\rho(x)$ lbs/ft^3.

(a) Write a formula to express the total mass of the material removed during drilling.

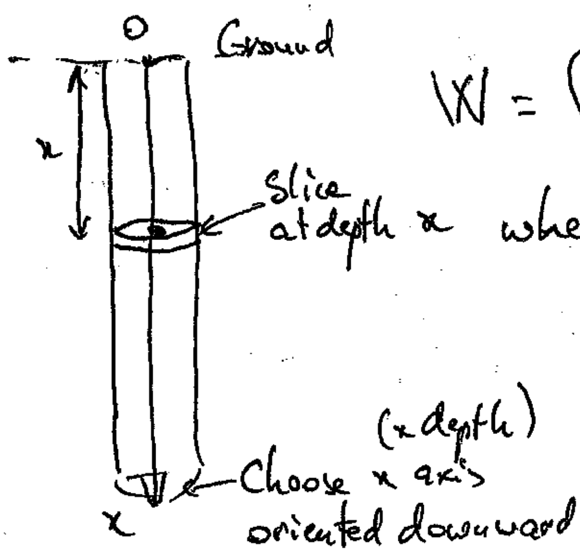
Since the density varies with depth, we should slice the pit with thin horizontal slices.

$$\text{mass} = m \approx \sum m_{slice} \approx \sum \rho(x) \cdot A_{slice} \cdot Th_{slice} \xRightarrow{\text{Take limit}} m = \int_{x=0}^{x=1000} \rho(x) \cdot \pi \cdot 1^2\, dx$$

so

$$\boxed{m = \pi \int_0^{1000} \rho(x)\, dx}$$ Answer for (a)

(b) Write a formula to express the total work done in removing the drilled material to the ground level.



$$W = \int_?^? dW$$

where dW = work to get the slice at depth x to the surface

$dW = \text{weight}_{slice} \cdot \text{dist}_{slice}$

$dW = \rho(x) \cdot V_{slice} \cdot \text{dist}_{slice} = \rho(x) \cdot A_{slice} \cdot \text{dist}_{slice} \cdot Th_{slice}$

but $A_{slice} = \pi \cdot 1^2 = \pi$ so

$$W = \int_{x=0}^{x=1000} \rho(x) \cdot \pi \cdot x\, dx = \boxed{\pi \int_0^{1000} x\rho(x)\, dx}$$