

Name: \_\_\_\_\_

Panther ID: \_\_\_\_\_

Exam 1

Calculus II

Spring 2016

**Important Rules:**

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (12 pts) (a) (4 pts) A particle moves on a straight line and let  $v(t)$  represent the velocity (in ft/second) of the particle at time  $t$  (in seconds), where  $t \in [0, 10]$ . Fill in the blanks with appropriate words.

Then  $v'(t)$  represents \_\_\_\_\_ ,

while  $\int_0^{10} |v(t)| dt$  represents \_\_\_\_\_ .

(b) (4 pts) Suppose that oil is leaking into the ocean from a damaged tanker at a rate of  $r(t)$  gallons per day, where  $t$  is the time in days since the accident occurred. In one sentence, explain what the integral  $\int_2^3 r(t) dt$  represents.

(c) (4 pts) Simplify as much as possible the expression

$$\frac{d}{dx} \left( \int_e^{e^x} (\ln t)^2 dt \right)$$

2. (8 pts) Use summation notation and then find the value of the sum:  $2+4+6+8+ \dots +2014+2016$

It's OK to leave your answer as a product.

3. (20 pts) True or False questions (4 pts each). In each case, circle your answer (2 pts) and briefly justify (2 pts).

(a) Any bounded sequence must be monotone. **True False**

**Justification:**

(b) If a sequence  $\{a_n\}$  is monotone and satisfies  $2 \leq a_n \leq 5$  for all  $n \geq 1$ , then  $\{a_n\}$  is convergent. **True False**

**Justification:**

(c) The sequence  $a_n = \frac{(-1)^n}{\sqrt{n}}$  is divergent. **True False**

**Justification:**

(d) The series  $\sum_{k=1}^{+\infty} \frac{1}{k}$  is convergent. **True False**

**Justification:**

(e) If  $\int_0^5 f(x) dx = 10$  and  $\int_3^5 f(x) dx = -3$  then  $\int_0^3 f(x) dx = 13$ . **True False**

**Justification:**

4. (8 pts) Show that the sequence  $a_n = \frac{5^n}{n!}$  is eventually monotone (and specify the type of monotonicity you find).

5. (16 pts) Evaluate each of the following series or show it diverges:

(a) 
$$\sum_{k=2}^{+\infty} (-1)^k \frac{2^{3k}}{3^{2k}}$$

(b) 
$$\ln\left(\frac{1}{3}\right) + \ln\left(\frac{3}{5}\right) + \ln\left(\frac{5}{7}\right) + \ln\left(\frac{7}{9}\right) + \dots$$

6. (8 pts) Find the average value of  $f(x) = \sqrt{x}$  on the interval  $[1, 9]$ .

7. (24 pts) Compute each integral and simplify your answer when possible (6 pts each):

(a)  $\int_0^2 |2x-3| dx$

(b)  $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx$

(c)  $\int_0^{\pi/4} 4 \sin(2x)(1+\cos(2x))^3 dx$

(d)  $\int_0^1 \frac{x}{1+3x^2} dx$

**8.** (12 pts) Choose ONE to prove. If possible, use sentences or formulas with complete justifications. The grading will be based on the clarity of your logic and explanations, as much as on any calculations involved.

(a) State and prove the geometric series theorem.

(b) State FTC, both parts. Prove the part of FTC about  $\frac{d}{dx}(\int_a^x \dots)$ . You may use without proof MVT for integrals.