Name: ______________________________ **Panther ID:** _______________

Exam 3 Calculus II Spring 2016

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.

2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.

3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.

4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (12 pts) Circle the correct answer. No justification is necessary for this problem (3 pts each).

(a) The partial fraction decomposition for $\frac{2x+5}{x^4+4x^2}$ is of the form:

(i) $\dfrac{A}{x^2} + \dfrac{B}{x^2+4}$ (ii) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+4}$ (iii) $\dfrac{2x}{x^2} + \dfrac{5}{x^2+4}$

(iv) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{x^4}$ (v) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+2} + \dfrac{D}{(x+2)^2}$

(b) For the integral $\int \sqrt{4x^2-9}\,dx$, the following substitution is helpful:

(i) $x = 3\sin\theta$ (ii) $w = 4x^2 - 9$ (iii) $2x = 3\sec\theta$ (iv) $2x = 3\tan\theta$ (v) $w = (2x-3)^2$

(Don't spend time evaluating the integral. It is not required.)

(c) Let $f(x)$ be a positive, increasing, continuous function on $[a, b]$ and let R_4 be the right end point Riemann sum approximation with 4 subdivisions of the integral $\int_a^b f(x)\,dx$. Then compared with the integral, R_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about f)

(d) Let $f(x)$ be a positive, concave down, continuous function on $[a, b]$ and let M_4 be the mid-point Riemann sum approximation with 4 subdivisions of the integral $\int_a^b f(x)\,dx$. Then compared with the integral, M_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about f)

2. (10 pts) In each case answer True or False. No justification needed. (2 pts each)

(a) $\sum_{k=1}^{+\infty} \frac{1}{k} = 0$. **True** **False**

(b) If $0 < a_k < \frac{1}{k^2}$ for all $k \geq 1$, then $\sum_{k=1}^{\infty} a_k$ is convergent. **True** **False**

(c) If $\lim_{k \to +\infty} a_k = 0$ then $\sum_{k=1}^{\infty} a_k$ is convergent. **True** **False**

(d) If $\sum_{k=1}^{\infty} |a_k|$ is convergent, then $\sum_{k=1}^{\infty} a_k$ is convergent. **True** **False**

(e) The series $2 - 1 - 1 + 2 - 1 - 1 + 2 - 1 - 1 + ...$ converges to 0. **True** **False**

3. (10 pts) Evaluate the improper integral or show it diverges $\int_0^{+\infty} e^{-2x}\, dx$

4. (14 pts) Use a trigonometric substitution to evaluate $\displaystyle\int \frac{1}{(4-x^2)^{3/2}}\, dx$

5. (24 pts) For each of the following series, determine if the series is absolutely convergent (AC), conditionally convergent (CC), or divergent (D). Answer **and carefully** justify your answer. Very little credit will be given just for a guess. Most credit is given for the quality of the justification. (12 pts each)

(a) $\frac{1}{2} - \frac{3}{2^2} + \frac{5}{2^3} - \frac{7}{2^4} + \frac{9}{2^5} - \frac{11}{2^6} + ...$

(b) $\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^2+1}}$

6. (14 pts) Use any method – definition or operations on familiar series – to find the Maclaurin series of the function $f(x) = \ln(1 + 2x)$. (Recall that the Maclaurin series is the same as the Taylor series at $x_0 = 0$.)

7. (14 pts) Find the interval of convergence (with endpoints) of the series $\sum_{k=1}^{\infty} \frac{1}{k \cdot 3^k} (x-1)^k$.

8. (12 pts) Choose ONE to prove:

(a) State and prove the k-th term divergence test for series. (You may ignore the inconclusive case.)

(b) State and prove the p-series test (using the integral test).