

NAME: \_\_\_\_\_

Panther ID: \_\_\_\_\_

Worksheet week 1 - MAC 2312, Spring 2016

1. (6 pts) Determine a formula for the general term  $a_n$  of the sequence and decide whether the sequence converges or diverges. Briefly justify each case.

(a)  $a_1 = 1, a_2 = -1, a_3 = 1, a_4 = -1, \dots$

(b)  $a_0 = 1, a_1 = -\frac{2}{3}, a_2 = \frac{4}{9}, a_3 = -\frac{8}{27}, \dots$

(c)  $a_1 = 1, a_2 = \frac{1}{2}, a_3 = 1, a_4 = \frac{2}{3}, a_5 = 1, a_6 = \frac{3}{4}, \dots$

2. (6 pts) In this problem you will prove (with Calculus) that the area of a circle of radius  $r$  is given by  $A = \pi r^2$ .

(a) Consider a regular pentagon inscribed in this circle and let  $A_5$  denote the area of this pentagon. Find a formula for  $A_5$  in terms of the radius  $r$  of the circle (of course, some factor involving  $\sin(\pi/5)$  will also appear).

*Hint:* Using the center of the circle, divide the pentagon into 5 congruent triangles.

(b) Consider now a regular polygon with  $n$ -sides inscribed in this circle and let  $A_n$  denote the area of this polygon. Following your reasoning in part (a), find a formula for  $A_n$  in terms of the radius  $r$  of the circle.

(c) Give an informal reason why  $A = \lim_{n \rightarrow +\infty} A_n$  and then compute the limit to get the famous  $A = \pi r^2$ .