NAME: Solution Key / Grading key Panther ID: ____________

Worksheet week 2 - MAC 2312, Spring 2016

1. (a) Find a simple closed form for the sum $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\ldots+\frac{1}{n^2-1}$

Hint: Check that $\frac{1}{k^2-1}=\frac{1}{(k-1)(k+1)}=\frac{1}{2}\left(\frac{1}{k-1}-\frac{1}{k+1}\right)$ and note that you get a telescopic sum.

(b) Use the result in part (a) to find

$$\lim_{n\to+\infty}\sum_{k=2}^{n}\frac{1}{k^2-1}$$

Note: This limit is, by definition, the series $\sum_{k=2}^{+\infty}\frac{1}{k^2-1}$.

Thus, you proved that the series above is convergent and you found its exact sum.

2. Applications of geometric series theorem.

(2.5 pts) (a) Find the sum of the series (if it exists) $2/3+4/9+8/27+16/81+\ldots$

(2.5 pts) (b) Find the sum of the series (if it exists) $\sum_{k=2}^{+\infty}\frac{(-3)^k}{2^{2k+1}}$

(2.5 pts) (c) Express the number 0.37777777... as a ratio of two integers.

(2.5 pts) (d) Express the number 0.13131313... as a ratio of two integers.

Note: Generalizing the ideas from (c) and (d), one can prove that any periodic number (that is, any number whose decimal digits repeat) is, in fact, a rational number.

(a) $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+\ldots=\underbrace{\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k}_{(1pt)}=\frac{2}{3}\cdot\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k-1}=\frac{2}{3}\sum_{\ell=0}^{\infty}\left(\frac{2}{3}\right)^{\ell}=$

geometric series with $r=\frac{2}{3}$

$=\frac{2}{3}\cdot\frac{1}{1-\frac{2}{3}}=2$ rest (1.5 pts)

So the series converges to $\boxed{2}$

(b) $\underbrace{\sum_{k=2}^{+\infty}\frac{(-3)^k}{2^{2k+1}}}_{(1pt)}=\sum_{k=2}^{\infty}\frac{(-3)^k}{2^{2k}\cdot 2}=\frac{1}{2}\sum_{k=2}^{\infty}\left(-\frac{3}{4}\right)^k=$

geom. series with $r=-\frac{3}{4}$, so $|r|<1$ thus the series is convergent

(rest 1.5 pts)

$=\frac{1}{2}\left(-\frac{3}{4}\right)^2\cdot\frac{1}{1+\frac{3}{4}}=\boxed{\frac{9}{56}}$

The series converges to $\frac{9}{56}$

c) $0.37777\ldots = 0.3 + 0.07 + 0.007 + \ldots$

$$= 0.3 + \frac{7}{10^2} + \frac{7}{10^3} + \frac{7}{10^4} + \ldots$$ getting this (1,5 pts)

$$= 0.3 + \frac{7}{10^2}\sum_{k=0}^{\infty}\frac{1}{10^k} = 0.3 + \frac{7}{100}\cdot\frac{1}{1-\frac{1}{10}}$$

$$= \frac{3}{10} + \frac{7}{\cancel{100}}\cdot\frac{\cancel{10}}{9} = \frac{3}{10} + \frac{7}{90} = \boxed{\frac{34}{90}}$$ rest (1 pt)

d) $0.131313\ldots = 0.13 + 0.0013 + 0.000013 + \ldots$

$$= \frac{13}{10^2} + \frac{13}{10^4} + \frac{13}{10^6} + \ldots = \sum_{k=1}^{\infty}\frac{13}{100^k}$$ (1,5 pts)

$$= \frac{13}{100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{13}{100}\cdot\frac{1}{\frac{99}{100}} = \frac{13}{99}$$ (1 pt)