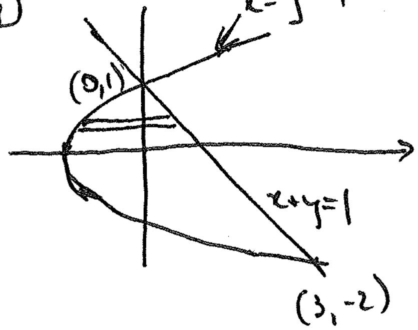
Worksheet week 6 - MAC 2312, Spring 2015 **NAME:** Solution Key

1. Find the area of the region bounded between $x = y^2 - 1$ and $x + y = 1$. Sketch and computation are required.

2. Set up an integral that gives the volume of the solid obtained when the region in problem 1 is rotated around the line $x = 3$. You are not required to evaluate the integral, but you should sketch the solid.

3. Set up an integral that gives the volume of the solid obtained when the region in problem 1 is rotated around the line $y = 1$. You are not required to evaluate the integral, but you should sketch the solid.

1



Intersection pts: $\begin{cases} x = y^2-1 \\ x+y=1 \end{cases} \Rightarrow y^2-1 = 1-y \Rightarrow y^2+y-2=0$

$(y+2)(y-1)=0$

so $y=-2$ or $y=1$

$(0,1)$ & $(3,-2)$

Easiest is to use horizontal strips.

$$A = \int_?^? \ell_{strip} \cdot Th_{strip}$$

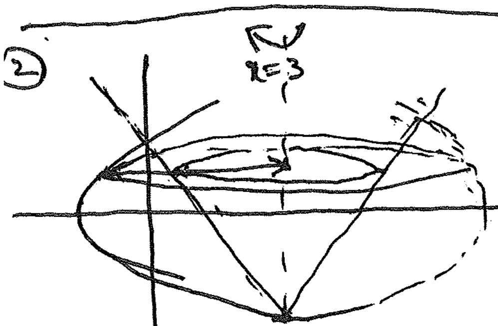
$Th_{strip} = dy$

$\ell_{strip} = x_2 - x_1 = (1-y)-(y^2-1)$

$\ell_{strip} = 2-y-y^2$

$$A = \int_{-2}^{1} (2-y-y^2)\,dy = \left(2y - \frac{y^2}{2} - \frac{y^3}{3}\right)\Bigg|_{y=-2}^{y=1} = \dots$$

2



Slicing method is best here.

$$V = \int_?^? A_{slice} \cdot Th_{slice} \qquad Th_{slice} = dy$$

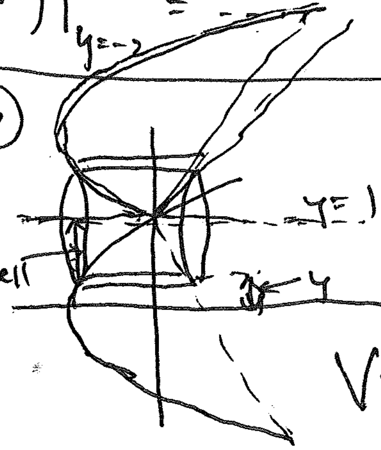
$A_{slice} = \pi\left(R_{outer}^2 - r_{inner}^2\right)$

$R_{outer} = 3 - x_{parab} = 3-(y^2-1) = 4-y^2$

$r_{inner} = 3 - x_{line} = 3-(1-y) = 2+y$

$$\boxed{V = \int_{y=-2}^{y=1} \pi\left[(4-y^2)^2 - (2+y)^2\right] dy}$$

3



Cylindrical shells is the best method here.

$$V = \int_?^? 2\pi R_{shell}\, h_{shell}\, Th_{shell}$$

$Th_{shell} = dy$

$h_{shell} = x_2 - x_1 = (1-y)-(y^2-1) = 2-y-y^2$

$R_{shell} = 1-y$

$$\boxed{V = \int_{y=-2}^{y=1} 2\pi(2-y-y^2)(1-y)\,dy}$$