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Quiz 1 - MAC 2312, Spring 2017

1. (3 pts) Is the statement below true or false? Answer (1 pt) and briefly justify your answer (2 pts).

Any monotone sequence is bounded. **True** **False**

Justification: An example is $a_n = n$ for $n \geq 1$. Clearly, $\{a_n = n\}_n$ is strictly increasing but not bounded (from above).

2. (3 pts) Find the limit of the sequence. If the limit does not exist or is infinite, explain the reason.

$\lim_{n\to+\infty} n^2e^{-n} = \lim_{n\to+\infty} \frac{n^2}{e^n} = 0$ since

$$\lim_{x\to+\infty} \frac{x^2}{e^x} \overset{\frac{\infty}{\infty}}{\underset{l'H}{=}} \lim_{x\to+\infty} \frac{2x}{e^x} = \lim_{x\to+\infty} \frac{2}{e^x} = 0$$

(OK also if you apply directly l'Hopital for the sequence, although, doing so, you tacitly assume that "n" becomes a real variable not just

3. (a) (2 pts) The first four terms of a sequence $\{a_n\}_{n=1}^{+\infty}$ are given below. Assuming the pattern continues, fill in expressions for the next term, a_5, and for the general term, a_n:

$a_1 = 2-\frac{\sqrt{1}}{1}$, $a_2 = 2+\frac{\sqrt{2}}{3}$, $a_3 = 2-\frac{\sqrt{3}}{5}$, $a_4 = 2+\frac{\sqrt{4}}{7}$, $a_5 = 2-\frac{\sqrt{5}}{9}$, ..., $a_n = 2+(-1)^n\frac{\sqrt{n}}{2n-1}$, ...

(b) (3 pts) Is the sequence $\{a_n\}_{n=1}^{+\infty}$ given in part (a) convergent? Answer and briefly justify.

Note that $\lim_{n\to+\infty} \frac{\sqrt{n}}{2n-1} = 0$ (by quick rule, or l'Hopital)

Thus $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty}\left(2+(-1)^n\frac{\sqrt{n}}{2n-1}\right) = 2$,

so the sequence $\{a_n\}_n$ is convergent to 2 (even though it oscillates around 2, the oscillations are getting smaller and smaller)