NAME: ______________________________ **Panther ID:** ____________

Worksheet week 1 - MAC 2312, Spring 2017

1. A basketball tossed straight up in the air reaches a high-point h_0 and falls to the floor. Each time the ball bounces to the floor it rebounds to a (constant) ratio r of its previous height ($0 < r < 1$). Assume the initial height is $h_0 = 20$ ft, and the ratio $r = 3/4$. Let h_n be the high-point of the ball after the n-th bounce.

(a) Find a recurrence relation and an explicit formula for the sequence $\{h_n\}_n$.

(b) What is the height of the ball after the 10th bounce? after the 20th bounce?

(c) What is the limit of the sequence $\{h_n\}_n$?

2. In this problem you will prove (with Calculus) that the area of a circle of radius r is given by $A = \pi r^2$.

(a) Consider a regular pentagon inscribed in this circle and let A_5 denote the area of this pentagon. Find a formula for A_5 in terms of the radius r of the circle (of course, some factor involving $\sin(\pi/5)$ will also appear).
Hint: Using the center of the circle, divide the pentagon into 5 congruent triangles.

(b) Consider now a regular polygon with n-sides inscribed in this circle and let A_n denote the area of this pentagon. Following your reasoning in part (a), find a formula for A_n in terms of the radius r of the circle.

(c) Give an informal reason why $A = \lim_{n\to+\infty} A_n$ and then compute the limit to get the famous $A = \pi r^2$.

3. Suppose your doctor prescribes a 100-mg dose of an antibiotic every 24 hours. Furthermore, the drug is known to have a half-life of 24 hours; that is, every 24 hours, half of the drug in your blood is eliminated. Let d_n be the amount of the drug in your blood after the nth dose.

(a) Find a recursive formula for this sequence. (*Hint:* Think how d_{n+1} is obtained in terms of d_n.)

(b) List the first 6 terms of the sequence $\{d_n\}_n$ and graph them.

(c) Take the limit in both sides of the recursive relation you found in part (a) to find the limit of the sequence $\{d_n\}_n$. What is the practical significance of this limit?

(d) Part (c) was (partly) based on the assumption that the sequence $\{d_n\}_n$ is convergent. Give an argument to show that the sequence $\{d_n\}_n$ is monotone and bounded and, hence, convergent.