NAME: ______________________ **Panther ID:** ____________

Worksheet week 9 - Trigonometric substitutions - MAC 2312, Spring 2017

Trigonometric substitutions are generally useful when dealing with integrals involving expressions of the type $\sqrt{a^2 - x^2}$, or $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$. The general idea of a trig. substitution is to use the basic trig. identities

$$\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta\ ,$$

to get rid of square-root in the integral.

General rules: (I wrote the first one, you should fill in the blanks for the other two)

• If the integrand involves $\sqrt{a^2 - x^2}$, then the substitution $x = a\sin\theta$ will usually be useful because

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\sin^2\theta} = \sqrt{a^2(1 - \sin^2\theta)} = \sqrt{a^2\cos^2\theta} = a\cos\theta\ .$$

• If the integrand involves $\sqrt{a^2 + x^2}$, then the substitution $x =$ ________ will usually be useful because

$\sqrt{a^2 + x^2} =$ ____

• If the integrand involves $\sqrt{x^2 - a^2}$, then the substitution $x =$ ________ will usually be useful because

$\sqrt{x^2 - a^2} =$ ____

Next, compute each of the integrals, by using the appropriate trigonometric substitution.

1. $\displaystyle\int \frac{dx}{x^2\sqrt{9 - x^2}}$

2. $\displaystyle\int \frac{dx}{(4x^2 - 1)^{3/2}}\ dx$

3. $\displaystyle\int_3^4 \frac{1}{\sqrt{x^2 - 6x + 10}}\ dx$ *Hint:* First, complete the square, then use a trig sub.

4. (bonus, if you have time) Earlier in the course we mentioned that it is not easy to compute the integral

$$\int \sqrt{a^2 - x^2}\ dx$$

Using a trig substitution, now you can compute it. Do so!

Hint: Along the way, you'll encounter the integral $\int \cos^2\theta\ d\theta$. The easiest way to compute this is to use the double angle identity $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$ and then integrate. However, don't forget that you still have to express your answer back in terms of x.