NAME: ______________________________ **Panther ID:** ____________

Exam 2 - MAC 2313 Fall 2018

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (15 pts) Given the function $f(x, y) = \ln(1 + x^2 + 3y^2)$ find:

(a) (5 pts) The partial derivatives f_x, f_y at an arbitrary point (x, y).

(b) (5 pts) The directional derivative of f at $P(0, 0)$ in the direction of $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j}$ (note that $\mathbf{a}$ is not a unit vector).

(b) (5 pts) A unit vector in the direction in which f increases most rapidly at $P(0, 0)$ and the rate of increase in this direction.

2. (20 pts) True or False questions. Circle your answer and give a brief justification (4 pts each).

(a) For any moving particle, the velocity vector and the unit tangent vector are parallel. **True** **False**

Justification:

(b) If $\frac{ds}{dt} = 3$ for all t, then $\mathbf{r}'(t) \perp \mathbf{r}''(t)$ for all t. **True** **False**

Justification:

(c) If $z = z(x, y)$ and $x = r\cos\theta, y = r\sin\theta$ then $\dfrac{\partial z}{\partial \theta} = -r\sin\theta + r\cos\theta$ **True** **False**

Justification:

(d) If $D = f_{xx}f_{yy} - (f_{xy})^2 < 0$ at a critical point P, then f has a relative extremum at P. **True** **False**

Justification:

(e) The function $f(x, y) = y^3$ has no absolute maximum on the region $x^2 + y^2 < 4$. **True** **False**

Justification:

3. (10 pts) Suppose that $p(x, y)$ denotes the atmospheric pressure at a point (x, y).
Given that $p(100, 98) = 1008$ mb (millibars), $p_x(100, 98) = -2$ mb/km and $p_y(100, 98) = 1$ mb/km, use local linear approximation to estimate the atmospheric pressure at the point $(103, 100)$.

4. (10 pts) Find the equation of the tangent plane to the ellipsoid $x^2 + 4y^2 + z^2 = 18$ at the point (1, 2, 1).

5. (10 pts) Find the curvature $\kappa(t)$ of the ellipse $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$, for $t \in [0, 2\pi]$. Use the formula

$$\kappa(t) = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}.$$

6. (10 pts) If the following limit exists, compute it. If the limit does not exist, justify why it doesn't.

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$$

7. (15 pts) At what point(s) on the circle $x^2 + y^2 = 1$ does the function $f(x, y) = xy$ have an absolute maximum value and what is that max ? Lagrange multipliers method is suggested, but a parametrization should also work.

8. (15 pts) Choose ONE proof. If you do both, only the larger score will be considered for this problem, but the second proof may give some bonus towards a previous problem where your score is smaller. You can use the back of the page.

(A) Find (with proof) the parametric equations of the projectile motion.

(B) Prove that for a differentiable function $f(x, y)$, the gradient is normal to the level curves of f.