Name: Solution Key   Panther ID:

Quiz 2   MAC-2313   Fall 2018

1. (3 pts) Match the following equations with the appropriate surface:

(i) $x^2 - 2y^2 - 3z^2 = 1$ (c)
(ii) $x^2 - 2y^2 - 3z^2 = 0$ (d)
(iii) $(x+1)^2 + 2(y-1)^2 + 3(z-2)^2 = 10$ (f)
(iv) $x - 2y^2 - 3z^2 = 1$ (e)
(v) $(x+1)^2 + 2(y-1)^2 - 3(z-2)^2 = 10.$ (b)
(vi) $x^2 + 3z^2 = 1$ (a)

(a) elliptic cylinder   (b) hyperboloid with one sheet   (c) hyperboloid with two sheets
(d) elliptic cone   (e) elliptic paraboloid   (f) ellipsoid

2. (8 pts) For both parts of this problem, consider the line L given by $x = 1 - 6t$, $y = 3 + 5t$, $z = 2 + 4t$, and the plane π given by $x + 2y - z = 1$.

(a) (4 pts) Determine if the line L intersects the plane π, is parallel to the plane π, or is contained in the plane π. Justify your answer.

Solution 1: Finding (possible) intersection of L and π is equivalent to solving the system

$$\begin{cases} x = 1-6t \\ y = 3+5t \\ z = 2+4t \\ x+2y-z=1 \end{cases}$$

Plugging in the first 3 eqns in the 4th get $(1-6t) + 2\cdot(3+5t) - (2+4t) = 1$

or $1 - \cancel{6t} + 6 + \cancel{10t} - 2 - \cancel{4t} = 1$

$5 = 1$ not possible, so the system has no solutions.

This means $\underline{L \parallel \pi}$.

Solution 2: Directional vector of the line L is $\vec{u} = \langle -6, 5, 4\rangle$

Normal vector of the plane is $\vec{n} = \langle 1, 2, -1\rangle$

Since $\vec{u}\cdot\vec{n} = -6 + 10 - 4 = 0$

$\vec{u} \perp \vec{n}$, so $\vec{u}$ is a vector parallel to π.

Next note that the point $(1,3,2)$ (which is on L) is not in π (since $1 + 2\cdot 3 - 2 \neq 1$),

so the line L is parallel to π.

(b) (4 pts) Find the equation of a plane $\tilde{\pi}$ which contains the given line L and is perpendicular to the given plane π.

Since $L \subset \tilde{\pi}$, its directional vector $\vec{u} = \langle -6, 5, 4\rangle$ is a vector parallel to $\tilde{\pi}$.

Since $\pi \perp \tilde{\pi}$, the normal $\vec{n}$ of π is also a vector parallel to $\tilde{\pi}$.

Thus $\vec{\tilde{n}} = \vec{n} \times \vec{u}$ is a normal vector for $\tilde{\pi}$

$$\vec{\tilde{n}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ -6 & 5 & 4 \end{vmatrix} = 13\vec{i} + 2\vec{j} + 17\vec{k}$$

Pick the point P_0 on L corresponding to $t=0$ $P_0(1,3,2)$

So $\tilde{\pi}$: $\boxed{13(x-1) + 2(y-3) + 17(z-2) = 0}$