NAME: _____ **Group Nr:** _____

Worksheet 09/06 - MAC 2313, F'18

1. Sketch the solid enclosed between $4x^2 + 4y^2 + z^2 = 5$ and $z = x^2 + y^2$ and describe their curve of intersection.

2. An airplane flies the route Miami-New Orleans. Your task is to approximate the **shortest** travel distance (which determines the necessary fuel, etc.). Use that the radius of Earth is $R = 6370$ km (and assume that the plane flies close to the surface of the Earth).

(a) First solve the problem assuming that Miami and New Orleans have the same latitude; so assume the geographical coordinates of the two cites are Miami (30° N, 80° W) and New Orleans (30° N, 90° W).

(b) Now solve the problem using the more precise geographical coordinates Miami (25.74° N, 80.19° W) and New Orleans (29.95° N, 90.07° W).

(c) More generally, if P_1, P_2 are two points on the same sphere of radius R and the spherical coordinates of the two points are $P_1(R, \theta_1, \phi_1)$, $P_2(R, \theta_2, \phi_2)$ describe a way to find the shortest distance on the sphere between P_1 and P_2.

Hint 1: On a sphere, the geodesics (paths of shortest distance) are arcs on great circles, that is circles obtained from the intersection of the sphere with planes through the center of the sphere.

Hint 2: Suppose the sphere has center at the origin O and two points P_1, P_2 are given on the sphere. The geodesic between P_1 and P_2 is the (smaller) arc obtained from cutting the sphere with the plane through O, P_1, P_2.

Hint 3: The angle $\angle(P_1OP_2)$ is important. Use the vectors $\mathbf{OP_1}$ and $\mathbf{OP_2}$ to find it.