NAME: ________________________________ **Panther ID:** ____________

Final Exam - MAC 2313 Spring 2012

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (20 pts) Write the defining formula for each of the following.

(a) Divergence of a vector field $\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + g(x, y, z)\mathbf{k}$;

(b) The speed of a particle whose position vector at time t is $\mathbf{r}(t)$;

(c) The arc length of a curve $\mathbf{r}(t)$, from $t = a$ to $t = b$;

(d) The Jacobian of a change of variables $x = x(u, v), y = y(u, v)$;

(e) The equation of the tangent plane at a point (x_0, y_0, z_0) of a surface given by the graph of $z = f(x, y)$.

2. (15 pts) (a) Find a unit vector in the direction in which $f(x, y) = y^3e^{2x}$ decreases most rapidly at the point $P(2, -1)$. Find that rate of decrease.

(b) Compute the directional derivative of f at P in the direction of the vector $\mathbf{a} = -4\mathbf{i} + 3\mathbf{j}$.

3. (15 pts) Given the points $A(1,1,0), B(0,1,1), C(1,0,1)$ in $\mathbf{R}^3$, find the following:

(a) (5 pts) The length of the segment $|AB|$.

(b) (5 pts) The angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(c) (5 pts) The area of the triangle ABC.

4. (20 pts) Let L be the line defined by the parametric equations

$$x = 1 - 2t, \quad y = 2 + 3t, \quad z = 3 + t,$$

and let P be the plane defined by $3x + 2y + z = 0$.

(a) (8 pts) Show that L and P are **NOT** perpendicular.

(b) (12 pts) Find an equation of the plane Q that both contains L and is perpendicular to P.

5. (15 pts) Find all critical points of the function $f(x, y) = -3x^4 + 2y^3 - 6x^2y - 144$. Classify the critical points as either relative minima, relative maxima or saddle points.

6. (15 pts) Evaluate the surface integral

$$\iint_{\sigma} z^2 \, dS, \quad \text{where } \sigma \text{ is the portion of the cone } z^2 = x^2 + y^2 \text{ between the planes } z = 1 \text{ and } z = 2.$$

7. (20 pts) (a) (14 pts) Show that the 2-dimensional vector field $\mathbf{F}(x, y) = 2xy\mathbf{i} + (x^2 - \sin y)\mathbf{j}$ is conservative everywhere and find a potential function.

(b) (6 pts) Using part (a), or by direct computation, find the work done by $\mathbf{F}(x, y)$ on a particle that moves along the line segment from $(0, 1)$ to $(1, 0)$.

8. (10 pts) Show that $\text{curl}(\nabla\phi) = \mathbf{0}$, where $\phi = \phi(x, y, z)$ is a function whose second order partial derivatives exist and are continuous.

9. (15 pts) Compute the integral

$$\int_C 3xy\ dx + 2x^2\ dy\ ,$$

where C is the counter-clock-wise oriented boundary of the region R shown in the picture. R is bounded above by the line $y = x$ and below by the parabola $y = x^2 - 2x$.

10. (15 pts) Find the coordinates of the centroid of the solid hemisphere $x^2 + y^2 + z^2 \leq a^2$, $z \geq 0$. You may use the symmetry of the region to simplify your computations.