

Name: Solution key

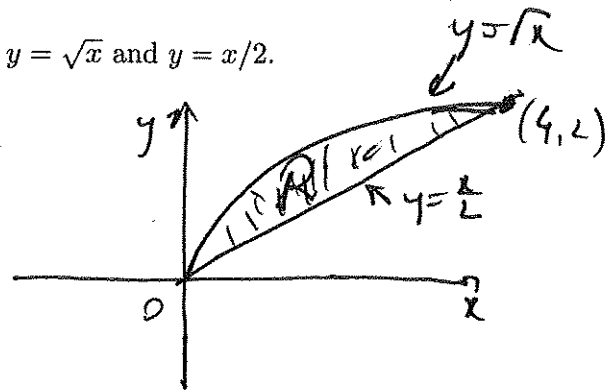
Panther ID: _____

Quiz 11/7 MAC-2313

Fall 2017

1. (6 pts) Let R be the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = x/2$.

(a) (2 pts) Sketch the region R in the xy -plane.



(b) (2 pts) Fill in the missing limits of integration:

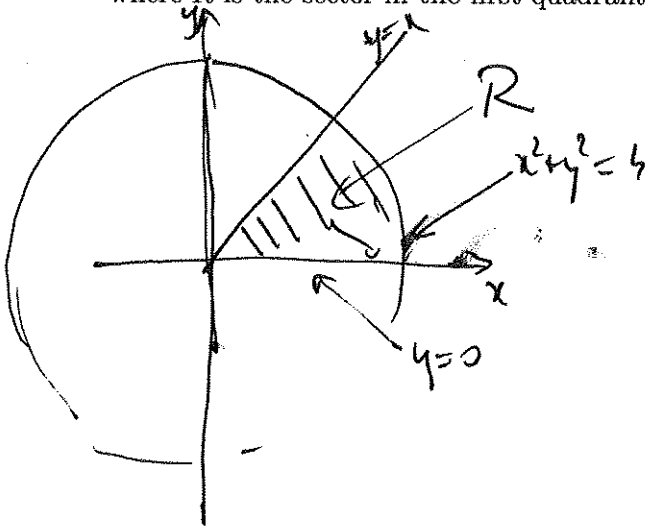
$$\int_R \int f(x, y) dA = \int_{x=0}^{x=4} \int_{y=x/2}^{y=\sqrt{x}} f(x, y) dy dx$$

(c) (2 pts) Fill in the missing limits of integration:

$$\int_R \int f(x, y) dA = \int_{y=0}^{y=2} \int_{x=y^2}^{x=2y} f(x, y) dx dy$$

2. (6 pts) Use polar coordinates to evaluate $\iint_R \frac{1}{1+x^2+y^2} dA$,

where R is the sector in the first quadrant bounded by $y = 0$, $y = x$ and $x^2 + y^2 = 4$.



$$\iint_R \frac{1}{1+x^2+y^2} dA =$$

$$= \int_{\theta=0}^{\theta=\frac{\pi}{4}} \int_{r=0}^{r=2} \frac{1}{1+r^2} r dr d\theta$$

$$= \int_{\theta=0}^{\theta=\frac{\pi}{4}} \left. \frac{1}{2} \ln(1+r^2) \right|_{r=0}^{r=2} d\theta$$

$$= \frac{1}{2} (\ln 5 - \ln 1) \cdot \frac{\pi}{4} = \frac{\pi \cdot \ln 5}{8}$$