Name: _____ **Panther ID:** _____

Exam 1 - MAC2311 Spring 2014

General Directions: *Read the problems carefully and provide answers exactly to what is requested. Use complete sentences and use notation correctly. Incomprehensible work is worthless. I am grading the work, not just the answer. Don't rush, don't try to do too many steps of a computation at once; work carefully. Good luck!*

1. (10 pts) Consider the function $f(x) = \sqrt{10 - 2x}$.

(a) (3 pts) Find the domain of f. Write your answer in interval form.

(b) (2 pts) Find the range of f. Write your answer in interval form.

(c) (5 pts) Find a formula for the inverse function $f^{-1}(x)$ and specify its domain.

2. (10 pts) (a) (5 pts) Solve for x: $\log_{10}(x - 14) - \log_{10} 2 = 3$

(b) (5 pts) Find an equivalent expression, without inverse trigonometric functions, for $\tan(\arccos x)$.

4. (20 pts) Find the following limits. If the limit is infinite or does not exist, specify so. (5 pts each)

(a) $\lim_{x\to 1} \dfrac{3x-3}{x^2+2x-3}$

(b) $\lim_{t\to 2^+} \dfrac{1-2t}{t-2}$

(c) $\lim_{t\to -\infty} \dfrac{2-t^3}{7t^3+t+3}$

(d) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

5. (14 pts) Compute each of the following limits:

(a) $\lim_{x\to 0} \dfrac{\sin^2(3x)}{x\tan(2x)} =$

(b) $\lim_{x\to +\infty} \left(x - \dfrac{x^2}{x+2}\right) =$

6. (10 pts) Let $P(t)$ represent the population in millions of a certain country at time t in years where $t = 0$ corresponds to year 2000. For parts (a) and (b), use one sentence to explain in practical terms what each equality is saying.

(a) (2 pts) $P(10) = 19.8$

(b) (3 pts) $P'(10) = -0.1$

(c) (5 pts) With the information from (a) and (b) estimate the population of this country this year. With one more sentence explain why your estimate may not be entirely accurate.

7. (10 pts) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{2x-1}$.

8. (10 pts) These are True or False questions. No justification required. No partial credit. 2 points each.

(a) For all $a > 0$, $\sqrt{a^2+4} = a+2$. **True** **False**

(b) If $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$, then f is continuous at $x = 2$. **True** **False**

(c) The equation $x^3 - 3x + 1 = 0$ has a solution in the interval $[0, 1]$. **True** **False**

(d) If f is continuous at $x = 2$, then f is differentiable at $x = 2$. **True** **False**

(e) If $\lim_{x\to 1} f(x) = 3$, then for $x \neq 1$ sufficiently close to 1, $f(x) < 3.1$. **True** **False**

9. (a) (3 pts) Write the ϵ-δ definition for $\lim_{x\to a} f(x) = L$.

Choose ONE of the parts (b) or (c) and circle the one you try. Only ONE will be graded. Note the different point values.

(b) (7 pts) Use the ϵ-δ definition to show that $\lim_{x\to 2}(5x-7) = 3$.

(c) (12 pts) Use the ϵ-δ definition to show that $\lim_{x\to 2}(5x^2-7) = 13$.