Name: ____________________________ **Panther ID:** ______________

FINAL EXAM Calculus I Fall 2013

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (25 pts) Compute y' (5pts each):

(a) $y = 3x^2 + 2^x - 3e^2$

(b) $y = \dfrac{\sin x}{x}$

(c) $y = \arcsin x - \sqrt{1+x^2}$

(d) $y = \sec(\tan(5x))$

(e) $y = (\cos x)^x$

2. (25 pts) Compute the following limits, **SHOWING YOUR WORK**. (5pts each)

(a) $\displaystyle\lim_{x\to 2^-} \frac{x^2-8}{x-2}$

(b) $\displaystyle\lim_{x\to -\infty} \frac{2x^5-3x^2+7x-2}{x^3-8}$

(c) $\displaystyle\lim_{x\to +\infty} x\sin(1/x)$

(d) $\displaystyle\lim_{x\to +\infty} \frac{\ln x}{x^2+2x+7}$

(e) $\displaystyle\lim_{x\to 0}(1-3x)^{1/x}$

3. (20 pts) Find each indicated antiderivative (5pts each):

(a) $\displaystyle\int (5e^x + 2\sqrt{x} - \sec^2 x)\, dx$

(b) $\displaystyle\int \frac{1}{x}(x^2 + x + 1)\, dx$

(c) $\displaystyle\int x\sin(7x^2)\, dx$

(d) $\displaystyle\int x\sqrt{x+4}\, dx$

4. (10 pts) Find the tangent line to the curve $2xy + \pi \sin y = 2\pi$ at $(1, \pi/2)$.

5. (14 pts) These are True or False questions. No partial credit. 2 points each.

a. A discontinuous function never has an absolute maximum. **True** **False**

b. If $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$, then f is continuous at $x = 2$. **True** **False**

c. If $f'(2) = 0$, then f has a relative maximum or minimum at $x = 2$. **True** **False**

d. To compute the derivative of $\sin(\ln x)$ we must use the product rule. **True** **False**

e. If f is differentiable at $x = 2$, then f is continuous at $x = 2$. **True** **False**

f. If $\lim_{x \to 1} f(x) = 2$, then for x sufficiently close to 1, $f(x)$ cannot equal 1.9. **True** **False**

g. If $\lim_{x \to 1} f(x) = 2$ and $f(1) = 4$, then for x sufficiently close to 1, $f(x) > 3.9$. **True** **False**

6. (12 pts) Sketch the graph of a function $f(x)$ satisfying ALL of the following conditions:

- $f(x)$ has domain $\mathbf{R}$ except $x = -2$ and $x = 2$ and is continuous everywhere except $x = \pm 2$;
- $f(x)$ is an even function;
- $\lim_{x \to 2} f(x) = +\infty$;
- $\lim_{x \to -\infty} f(x) = 3$;
- The graph has only one critical point and that is a local minimum at $(0, 5)$;
- $f''(x) > 0$ for all values of x in the domain.

7. (10 pts) (a) Write the definition with limit of the derivative.

For each of the following, fill in the blanks with the most appropriate words or expressions:

(b) If $f'(x) = 0$ for all $x \in (a, b)$, then $f(x)$ is ________________________ on the interval (a, b).

(c) The *instantaneous* rate of change of a function $f(t)$ at $t = 2$, is given by ________________________.

(d) A polynomial function of degree 4 can have at most ______ inflection points.

(e) If $f'(x_0) = 0$ and $f''(x_0) < 0$, then x_0 is a ________________________ for the function.

8. (10 pts) Using calculus, find the dimensions of a rectangular packaging box, made of cardboard, that satisfies all of the following requirements: (a) the volume of the box is 9000 cm^3; (b) the base (and top) of the box must be a rectangle whose length is twice its width. (c) the amount of cardboard used to make the box should be minimal.

9.(20 pts) The steps of this problem should lead you to a complete graph of the function $f(x) = x^2e^x$. Where indicated, work should be shown below, or on the back of the paper.

(a) (1 pts) The domain of this function is ________________

(b) (4 pts) The derivative (in factored form) is $f'(x) =$ ________________ . Show work.

(c) (3 pts) Critical points of f are: ________________ . Show work.

(d) (3 pts) Do a sign chart for f' and mark the intervals where f is increasing, respectively decreasing.

(e) (4 pts) End behavior: $\lim_{x \to +\infty} x^2e^x =$ ______ $\lim_{x \to -\infty} x^2e^x =$ ______ Show work below .

(f) (5 pts) Using all the previous steps, sketch the graph of $f(x) = x^2e^x$. Label on the graph the coordinates of critical points and specify their type (relative/absolute min/max).

Bonus 2pts: I did not ask you to do the analysis of the second derivative. Without computing the second derivative, how many inflection points do you expect?

10. (10 pts) (a) (6 pts) Find the linear approximation of the function $f(x) = \sqrt[3]{x}$ at $x = 27$.

(b) (4 pts) Use part (a) to approximate $\sqrt[3]{27.12}$ without using a calculator.

11. (7 pts) Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 2π mi^2/h. At what rate is the radius of the spill increasing when the radius is 2 miles?