

PREREQUISITES FOR MAC 2311

Here is a summary of important notions you should know from precalculus:

ALGEBRA

1. Factoring, algebraic formulas, completing the square

$$x^2 - y^2 = (x - y)(x + y), \quad (a \pm b)^2 = a^2 \pm 2ab + b^2, \\ x^3 + y^3 = (x + y)(x^2 - xy + y^2), \quad x^3 - y^3 = (x - y)(x^2 + xy + y^2), \quad \text{etc.}$$

2. Exponents:

$$x^n x^m = x^{n+m}, \quad \frac{x^n}{x^m} = x^{n-m}, \quad x^{-n} = \frac{1}{x^n}, \\ x^n y^n = (xy)^n; \quad x^{\frac{1}{n}} = \sqrt[n]{x}, \quad \text{etc.}$$

3. Fractions:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}, \quad \text{etc.}$$

4. Equation solving: finding solution for x if

$$ax + b = 0, \quad ax^2 + bx + c = 0 \text{ by factoring or with quadratic formula, etc}$$

5. Inequalities and absolute values:

$$|x| < a \leftrightarrow -a < x < a; \quad |x| \geq a \leftrightarrow (x \geq a \text{ or } x \leq -a)$$

6. Basic functions – their domains and graphs:

$$f(x) = x, \quad f(x) = x^2, \quad f(x) = x^3, \quad f(x) = |x|, \quad f(x) = \sqrt{x}, \quad f(x) = 1/x, \\ f(x) = b^x, \quad f(x) = \log_b x \text{ (where } b > 0, b \neq 1\text{); in particular } f(x) = e^x, \quad f(x) = \ln x, \\ f(x) = \sin x, \quad f(x) = \cos x, \quad f(x) = \tan x.$$

7. Find the domain, axis intercepts for simple functions: e.g.

$$f(x) = \sqrt{2x + 3}, \quad f(x) = \sqrt{4 - x^2}, \quad f(x) = \frac{x + 3}{x^2 - 1}$$

8. Logarithms – definition and properties

$$\log_b y = x \text{ if and only if } b^x = y \\ \log(xy) = \log x + \log y, \quad \log\left(\frac{x}{y}\right) = \log x - \log y, \quad \log(x^p) = p \log x$$

GEOMETRY and TRIGONOMETRY

1. Pythagorean theorem: $b^2 + c^2 = a^2$, where b, c are the sides of a right triangle and a its hypotenuse.
2. Basic geometric formulas: e.g. area of a triangle, area of a rectangle, circumference of a circle, area of a circle, volume of a cylinder, volume of a sphere, etc.
3. Facts about congruent and similar triangles
4. Facts about lines – equations, slope given two points, point-slope formula, write the equation of a line given two points, slopes of parallel/perpendicular lines

$$ax + by + c = 0, \quad y = mx + b, \quad m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \quad y - y_0 = m(x - x_0),$$

parallel lines have equal slopes; perpendicular lines have slopes which are negative reciprocal, i.e. $m_2 = -1/m_1$.

5. Distance formula; equation of a circle

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \quad (x - a)^2 + (y - b)^2 = r^2 \quad \text{-- circle with center } (a, b) \text{ and radius } r$$

6. Basic trigonometric identities

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 & \sec^2 \theta &= 1 + \tan^2 \theta & \csc^2 \theta &= 1 + \cot^2 \theta \\ \sin(-\theta) &= -\sin \theta & \cos(-\theta) &= \cos \theta & \tan(-\theta) &= -\tan \theta \\ \sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta & \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta \end{aligned}$$

7. Trigonometric Values (no calculators allowed on exams): e.g. you should be able to determine, using the unit circle and your knowledge of "nice" angles, the exact values of $\sin\left(\frac{7\pi}{6}\right)$ or $\cot\left(\frac{3\pi}{4}\right)$.

8. Sum and difference formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B, \quad \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

9. Double-angle formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta, \quad \cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

10. Half-angle formulas

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}, \quad \cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

11. Solving simple trig. equations – e.g. $\tan \theta = -\sqrt{3}$, $\cos(2\theta) = \sin \theta$.

12. Law of Sines; Law of Cosines in a triangle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}, \quad a^2 = b^2 + c^2 - 2bc \cos A, \quad \text{triangle with sides } a, b, c \text{ and angles } A, B, C, \text{ respectively}$$