

1. Using a sign chart for the derivative determine the intervals where the function $f(x) = x^2e^{-x}$ is increasing and the intervals where is decreasing. Determine the critical points and their nature. Determine the end behavior of the function and then sketch its graph.

2. (This problems helps you find on your own the proof of MVT) Recall MVT: Assume that $f(x)$ is a continuous function on a closed interval $[a, b]$ and assume that f is differentiable for all $x \in (a, b)$. Then there exists (at least) a point $c \in (a, b)$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a} .$$

Do the following steps to prove MVT.

- (a) Draw a picture to illustrate MVT geometrically.
- (b) Write the equation of the secant line connecting the points $(a, f(a))$ and $(b, f(b))$.
- (c) Find a formula for the function $h(x)$ which measures the vertical distance between the y coordinate of the point on the secant line and the y coordinate of the point on the graph of f for an arbitrary value $x \in [a, b]$.
- (d) Show that the function $h(x)$ satisfies the assumptions of Rolle's theorem for $x \in [a, b]$.
- (e) Write the conclusion of Rolle's theorem applied to h and show that it translates exactly in the conclusion of MVT for f .