NAME: _____ **Panther ID:** _____

Worksheet week 7 - MAC 2311, Spring 2014

1. Many population growth/decay models follow an exponential model. An exponential model is characterized by the property that the rate of change of the population is proportional to its size. Let $P(t) = P_0 e^{kt}$ be a certain population at time t, where P_0 and k are parameters.

(a) What is the meaning of P_0?

(b) Show that $P(t) = P_0 e^{kt}$ satisfies $P'(t) = kP(t)$, so, indeed, the rate of change of the population is proportional to its size, k being the constant of proportionality.

2. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

(a) Find an expression for the number of bacteria after t hours.

(b) Find the number of bacteria after 3 hours.

(c) Find the rate of growth after 3 hours.

(d) When will the population reach 10,000?

3. A curve passes through the point $(0, 7)$ and has the property that the slope of the curve at every point P is three times the y-coordinate of P. What is the equation of the curve?

4. Suppose there is a rail-road track along the graph of $f(x) = 2^x$. Let's call it the 2^x-track to infinity! Suppose you live in a city located at the origin $(0, 0)$, so your city is not on the track to infinity. You want to remedy this situation, so you plan to built a railroad from your city to connect smoothly with the 2^x-track to infinity. The problem is that your city can only produce *straight* railroad track. Can you build a straight railroad so that you take a train from your city at $(0, 0)$, you get to some point on $y = 2^x$ and from there the train shifts smoothly onto the 2^x-track to infinity? Be careful, you don't want the train to derail at the connection!