NAME: ―――――――――――――――――――――――――― **Panther ID:** ―――――――――――

Spring Break Worksheet – due Wednesday, March 19 - MAC 2312, Spring 2014

1. (a) Use IBP to derive a reduction formula for

$$\int x^n e^{-wx}\, dx, \text{ where } n \text{ is a nonnegative integer and } w \text{ is a positive constant.}$$

Next, you will use your reduction formula in part (a) to show that

$$\int_0^{+\infty} x^n e^{-wx}\, dx = \frac{n!}{w^{n+1}}\ .$$

Follow these steps:

(b) Denote $I_n = \int_0^{+\infty} x^n e^{-wx}\, dx$. Compute directly $I_0 = \int_0^{+\infty} e^{-wx}\, dx$.

(c) Use l'Hopital to show that $\lim_{x \to +\infty} x^n e^{-wx} = 0$.

(d) Use the reduction formula from part (a) and the observation in (c), to get the recursive formula

$$I_n = \frac{n}{w} I_{n-1}, \text{ for all } n \geq 1.$$

(e) From (d) and (b), conclude that $I_n = \frac{n!}{w^{n+1}}$.

Note: You have to trust me that the improper integral you computed is an important one. Hence, it was worth the effort!