Name: _____ **Panther ID:** _____

Exam 1 Calculus II Spring 2014

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (12 pts) Circle the correct answer for (a) and (b). Fill in the answer for (c).

(a) The average value of the function $f(x) = 1/x$ on the interval $[1, 3]$ is

(i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) $\frac{\ln 3}{2}$ (iv) $-\frac{1}{3}$ (v) none of the previous

(b) The expression

$$\frac{d}{dx}\Big(\int_0^{x^2} e^{t^2}\, dt\Big) \text{ is equivalent to}$$

(i) e^{x^2} (ii) e^{t^2} (iii) $2xe^{x^4}$ (iv) e^{x^4} (v) $e^{x^4} - 1$

(c) A water-tank, initially full, starts being drained at the moment $t = 0$. Suppose that $r(t)$ (in gals/min) is the rate of the water flow out of the reservoir at the moment t.

In one sentence, explain what the equality $\int_0^{30} r(t)dt = 1000$ says in practical terms.

2. (12 pts) Compute the area of the region bounded by the parabola $y = 4 - x^2$ and by the line $y = 3x$. Sketch of the region and full computation of the area are required for full credit.

3. (10 pts) (a) (5 pts) Find the area below $y = \sin x$ and above the x-axis, for $x \in [0, \pi]$.

(b) (5 pts) On a graph of $y = \sin x$, for $x \in [0, \pi]$, draw and shade the approximation of the area in part (a) given by the right end-point Riemann sum with 4 equal subdivisions. Is it an over-estimate or an under-estimate of the area? Briefly justify.

4. (28 pts) Compute each integral (7 pts each):

$(a) \int_0^2 \sqrt{4 - x^2}\, dx$

$(b) \int_0^1 \frac{1}{2x + 1}\, dx$

(c) $\displaystyle\int_0^{\sqrt{\pi}/2} x\sec^2(x^2)\,dx$

(d) $\displaystyle\int_{-4}^{0} x\sqrt{1-2x}\,dx$

5. (10 pts) Let $F(x) = \int_0^x f(t)\,dt$, for $x \in [0, 8]$ where f is the function whose graph is shown below.

(a) (6 pts) Find $F(0), F(3), F(5)$.

(b) (4 pts) Where does F have its absolute maximum value in the interval [0,8]? Its minimum value?

6. (10 pts) A stone dropped from the top of a building hits the ground with a speed of 96 ft/s. How tall is the building? Assume the initial velocity of the stone to be 0 and assume the gravitational acceleration $g = 32$ ft/s^2.

7. (16 pts) Set up integrals to represent each of the following (you **do not** have to evaluate).

(a) (8 pts) The volume of the solid generated when the region in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved around the y-axis. (Sketch of solid is required.)

(b) (8 pts) The volume of the solid when the region in the first quadrant enclosed by $y = \sqrt{1 - x^2}$ is rotated around the line $y = 1$. (Sketch of solid is required.)

8. (12 pts) (a) (4 pts) State the Fundamental Theorem of Calculus, both parts.

(b) (8pts) Choose ONE to prove:

(i) FTC part (b) – you may assume MVT for integrals, but specify when you are using it;

or

(ii) FTC part (a) – you may assume FTC part (b).