

Name: _____

Panther ID: _____

Exam 3

Calculus II

Spring 2014

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (10 pts) In each case answer True or False. No justification needed. (2 pts each)

(a) The alternating harmonic series is conditionally convergent.

(b) If $0 < a_k < \frac{1}{k}$ for all $k \geq 1$, then $\sum_{k=1}^{\infty} a_k$ is convergent.

(c) The sequence $a_n = \sqrt{n} - 1000$, $n \geq 1$ is eventually positive.

(d) $1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \frac{1}{4} + \dots = 1$

(e) $1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \dots = 1$

2. (12 pts) Sketch the graph of the cardioid $r = 2 + 2 \cos \theta$ by giving the coordinates of at least 4 points and then find the area bounded by the cardioid in the second quadrant.

3. (20 pts) Evaluate (or show it diverges) (5 pts each)

(a) $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

(b) $\lim_{k \rightarrow +\infty} \left(\frac{2}{3^k} - \frac{3}{2^k} \right)$

(c) $\sum_{k=0}^{+\infty} \left(\frac{2}{3^k} - \frac{3}{2^k} \right)$

(d) $\ln(1/3) + \ln(3/5) + \ln(5/7) + \ln(7/9) + \dots$

4. (24 pts) For each of the following series, determine if the series is divergent (D), conditionally convergent (CC), or absolutely convergent (AC). Answer **and carefully** justify your answer. Very little credit will be given just for a guess. Most credit is given for the quality of the justification. (8 pts each)

(a) $\frac{1}{3} - \frac{3}{5} + \frac{5}{7} - \frac{7}{9} + \frac{9}{11} - \frac{11}{13} + \dots$

(b) $\sum_{k=0}^{\infty} (-1)^k \frac{(k!)^2}{(2k+1)!}$

(c) $\sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k}$

5. (12 pts) Find the Taylor series at $x_0 = 1$ of the function $f(x) = \frac{1}{x}$.
(Your final answer should use the summation notation.)

6. (14 pts) Find the interval of convergence (with endpoints) of the series $\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$.

7. (12 pts) Choose ONE to prove:

(a) State and prove the divergence test for series.

(b) State and prove the convergence part of the simple comparison test (that is, the part saying that if a series converges then the other series converges).

8. (10 pts) Consider the sequence defined recursively by $a_0 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$, for $n \geq 0$. Show that a_n is convergent and find its limit.