Worksheet week 11 MAC-2312 – Spring 2014 **Name:** ______________________________

1. Among other things, in this exercise you'll prove $\sum_{k=1}^{\infty} 1/k = +\infty$.

(a) Let $n \geq 2$ be an integer.
Find the area below the graph of $f(x) = 1/x$
and above the x-axis on the interval $[1, n]$.
(your answer will, of course, depend on n).

(b) Shade the left endpoint Riemann sum
approximation of the area in part (a)
for the division of the interval $[1, n]$
into sub-intervals of length 1.
Write the expression for this Riemann sum.

(c) Explain why from parts (a) and (b) you obtain the inequality

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ ... \ + \frac{1}{n-1} \geq \ln n \ , \text{ and further, from this, } \sum_{k=1}^{+\infty} \frac{1}{k} = +\infty \ .$$

In the remaining parts of this exercise you'll show that the sequence

$$a_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n-1} - \ln n \text{ is convergent when } n \to \infty.$$

(The limit is the so called Euler's γ constant, $\gamma \approx 0.5772....$)

(d) Relating to parts (a) and (b) of the exercise, determine what a_n represents geometrically. If you understand this, you should be able to see that $a_n > 0$, for all $n \geq 2$ and that the sequence $\{a_n\}$ is strictly increasing. Briefly explain why these statements are true.

(e) Use the right-hand Riemann sum to show that $a_n < 1$, for all $n \geq 2$.

(f) From parts (c) and (d) it follows that the sequence $\{a_n\}$ is convergent. In one sentence explain why.

2. For each of the following series, first write them using summation notation, then find a closed form for the n-th term S_n of the sequence of partial sums and then finally determine whether the series converges:

(a) $\ln \frac{1}{2} + \ln \frac{2}{3} + \ln \frac{3}{4} + \ln \frac{4}{5} + \ldots$

(b) $\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \ldots$

Note: Both series are what we call "telescopic series".