Worksheet week 12 - MAC2312 Fall 2012 NAME: Answer Key.

1. Decide if each of the following series is convergent or divergent. Specify which test you are using and show how the test applies.

(a) $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2}$

convergent series
by simple (or limit) comparison test
compare with $\sum \frac{1}{4k^2}$

(b) $\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^3+1}}$

divergent series
Limit comparison test
with $\sum \frac{1}{\sqrt{k}}$

(c) $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

convergent series
by Ratio Test

$\rho = \lim_{k\to\infty} \frac{((k+1)!)^2}{(2k+2)!} \cdot \frac{(2k)!}{(k!)^2}$

$\rho = \lim_{k\to\infty} \frac{(k+1)^2}{(2k+1)(2k+2)} = \frac{1}{4} < 1$

2. Find the values of p for which the series is convergent $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$

Apply Integral Test:
If $p \leq 1$ integral & series diverge; If $p > 1$, integral & series converge.

3. True or False. Answer and briefly justify in each case.

(a) If $S_n = \sum_{k=1}^{n} a_k$ and $\lim_{n\to\infty} S_n$ does not exist or is not finite, then $\sum_{k=1}^{\infty} a_k$ is a divergent series.

True. By definition, $\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} S_n$

(b) If $\{a_k\}_k$ is a convergent sequence then the series $\sum_{k=1}^{\infty} a_k$ is also convergent.

False, if $a_k = \frac{1}{k}$, then $a_k \to 0$, but $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{1}{k} = +\infty$

(c) If $\{a_k\}_k$ is a divergent sequence then the series $\sum_{k=1}^{\infty} a_k$ is also divergent

True, if $\{a_k\}_k$ divergent $\Rightarrow \lim_{k\to\infty} a_k \neq 0$, so $\sum_{k=1}^{\infty} a_k$ diverges by divergence test.

(d) If $\sum_{k=1}^{\infty} a_k$ converges to A and $\sum_{k=1}^{\infty} b_k$ converges to B, then $\sum_{k=1}^{\infty}(a_k - b_k)$ converges to $A - B$.

True $\sum_{k=1}^{n}(a_k - b_k) = \left(\sum_{k=1}^{n} a_k\right) - \left(\sum_{k=1}^{n} b_k\right)$ and take the limit as $n \to +\infty$.

(e) If $a_k > 0$ for all k and $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty}(a_k)^2$ also converges.

True. If $\sum_{k=1}^{\infty} a_k$ converges $\Rightarrow \lim_{k\to+\infty} a_k = 0$ (div. test)

So, for a suf. large k, $a_k < 1$.

Thus $0 < a_k^2 < a_k$, for $k \geq k_0$

By simple comparison, since $\sum_{k=1}^{\infty} a_k$ converges $\Rightarrow$

$\Rightarrow \sum_{k=1}^{\infty} a_k^2$ converges too.