Name: ____________________________ **Panther ID:** ______________

Worksheet - Sep. 1 MAT 3501 Fall 2016

1. In each case, prove using induction:

(a) For all $n \geq 0$, $3^{2n+1} + 2^{n+2}$ is divisible by 7.

(b) (pb. 14 section 8.3) Show that the sum of the interior angles of a convex polygon is $180(n-2)$ degrees, where n is the number of sides of the polygon.

(c) For all $n \geq 1$, $1^2 + 2^2 + 3^2 + \ \dots\ + n^2 = \frac{n(n+1)(2n+1)}{6}$.

2. (Similar to pb. 4 in 2.4) (a) Find the prime decomposition of each of the numbers 20, 24, 120.

(b) List all divisors (factors) of 120 (you should have 16 of them). Find a good way to count them. Hint: A divisor of 120 has to be of the form $2^{k_1} \cdot 3^{k_2} \cdot 5^{k_3}$. What are the values that k_1, k_2, k_3 can have?

(c) Prove that if the prime decomposition of the number N is $N = p_1^{n_1} \cdot p_2^{n_2} \cdot \ \dots\ \cdot p_s^{n_s}$, then the number of divisors of N is $(n_1+1)(n_2+1)\ \dots\ (n_s+1)$.

3. Find a number that when divided by 2 is a perfect square, when divided by 3 is a perfect cube, and when divided by 5 is a perfect fifth power. (Hint: Use the prime factorization thm.)