Name: ————————————————   **Panther ID:** ——————————

Worksheet - Sep. 29   MAT 3501   Fall 2016

1. A farmer has a fixed length of p feet of fencing available. He would like to enclose a rectangular field with one side along a straight channel. He will not use fence for the side along the river, but just for the other three sides. He would like to enclose the maximum possible area. How should he do it?

Do the problem, in two different ways:
(i) Using completion of the square (so just simple algebra);
(ii) Using Calculus.

Additional Question: If the farmer drops the restriction that the shape of the field be rectangular and he considers any shape with a boundary along the straight channel, what do you think is his best option?

2. For a quadratic function $f(x) = ax^2 + bx + c$, do each of the following in two different ways:
(i) Using completion of the square (so just simple algebra);
(ii) Using Calculus.

(a) Find the coordinates of the vertex.
(b) Explain the rule: if $a > 0$ then the graph is a parabola concave up, if $a < 0$ then the graph is a parabola concave down.

3. (a) Use the fundamental theorem of algebra to show that any polynomial with real coefficients can be decomposed as a product of linear or quadratic factors, all with real coefficients.

(b) You mention the above result to your students and one comes up and says: "I thought that a polynomial like $x^4 + 1$ cannot be factored." How do you answer?

(c) Find a factoring as in part (a) for the polynomial $p(x) = x^3 + 2$.

4. Suppose that $p(x)$ is a polynomial of odd degree with real coefficients. Show that the equation $p(x) = 0$ must have at least one real real solution.