Name: ____________________ **Panther ID:** ____________

Worksheet - Oct. 27 MAT 3501 Fall 2016

1. We saw last time in class that any triangle has a (uniquely determined) circle that contains all of its vertices, the circumcircle. Thus, any triangle can be inscribed in a circle.

(a) Can ANY convex quadrilateral be inscribed in a circle? Briefly justify. (Think of three vertices of the quadrilateral being fixed and the fourth vertex mobile. You can do this with GeoGebra. See Pb. 3)

(b) Convex quadrilaterals that can be inscribed in a circle are also called *cyclic* quadrilaterals. Show that if a quadrilateral is cyclic then each pair of opposite angles has a sum of $180°$ (i.e. in a cyclic quadrilateral opposite angles are supplementary).

(c) The converse of statement in part (b) is also true and it is the main tool of showing that a quadrilateral is cyclic. Show that if in a convex quadrilateral the opposite angles are supplementary, then the quadrilateral is cyclic.

Hint: Do this by contradiction. Consider the triangle determined by three of the vertices, its circumcircle, and assume that the fourth vertex is not on this circle.

(d) Use (c) to show that a parallelogram is cyclic if and only if it is a rectangle.

(e) Use part (c) to show that a trapezoid is cyclic if and only if it is an isosceles trapezoid.

2. James Bond is at the southernmost point of a circular lake with diameter of two miles. He needs to get to the northernmost point (diametrally opposite). It is known that James Bond runs twice as fast as he can swim. He can swim directly across the lake, he can run all the way around the lake, or he can try a combination of swimming and running. What path should James Bond take in order to minimize the time of the trip? Use geometry and calculus to justify your answer.

3. Important take home problem (20 points value): Due Thursday, Nov. 3.
(a) If don't have this already, go to https://www.geogebra.org/ and download GeoGebra in your computer. Play a bit with its features.

(b) Construct, using GeoGebra, the incircle of a given triangle (you should start with the vertices, do the logical steps to get the incenter, and the last step should be the drawing of the incircle). Submit a printout of your work.

(c) Construct, using GeoGebra, the circumcircle of a given triangle. (There is a direct tool for this, but for your practice start again with the vertices and do the rest of the steps. Moreover. the direct tool will not show you directly the position of the circumcenter.) Submit a printout of your work. Next, use the "move" feature of the program (the arrow on the left) to move one of the vertices of the triangle keeping the other two vertices fixed. Notice what happens with the position of the circumcenter versus the triangle. Formulate a conjecture of the type: For "these" triangles the cicumcenter will lie inside the triangle, for "these" triangles the circumcenter will lie outside of the triangle and for "these" triangles the cicumcenter will lie on one side the triangle (be even more specific in this case). You'd get bonus point if you prove your conjecture.

(d) A *median* in a triangle is the segment that joins one vertex with the midpoint of the opposite side. With GeoGebra start with with a triangle and draw TWO of the medians. There is a "midpoint" feature that you should use. Let G denote the point of intersection of these two medians. Use distances and the move feature in GeoGebra to investigate the ratio (distance to vertex)/(distance to midpoint) that G determines on each median. Use similarity to prove this pattern. Next, draw a picture with all three medians. Prove that the three medians are concurrent. Their intersection point, traditionally denoted by G, is the centroid (or center of mass) of the triangle.