Name: ______________________________ **Panther ID:** _______________

Exam 1 MAT 3501 Fall 2017

1. (20 pts) For each of the following, answer if the statement is True or False. Then give a one line justification of your answer. (2 pts answer, 3 pts justification)

(a) When you give a proof by contradiction, you must contradict something that is given. **True** **False**

Justification:

(b) For any positive integer n, $\text{lcm}(n, n+1) = n(n+1)$. **True** **False**

Justification:

(c) For any positive integer n, the expression $n^2 + n + 41$ is a prime number. **True** **False**

Justification:

(d) If p and q are both prime numbers greater than 2, then $pq + 1$ is not prime. **True** **False**

Justification:

2. (14 pts) Given that two of the roots are rational, find **all** roots (real or complex) of the equation

$$2x^4 + 3x^3 + 2x^2 - 1 = 0 \ .$$

3. (14 pts) Prove (by induction, or otherwise) that $1^2 + 2^2 + 3^2 + \ \dots \ + n^2 = \frac{n(n+1)(2n+1)}{6}$, for any $n \geq 1$.

4. (14 pts) Prove that for any positive integers a, b, c, d the polynomial $x^{4a+3} + x^{4b+2} + x^{4c+1} + x^{4d}$ is divisible by $x^3 + x^2 + x + 1$. *Hint:* $x^3 + x^2 + x + 1 = (x + 1)(x^2 + 1)$.

5. (14 pts) Describe, with proof, the set of all positive integers a with the property that $\log_a 2017$ is a rational number. (*Hint:* Recall that 2017 is a prime number.)

6. (24 pts) Choose TWO of the following three (12 pts each)

(A) State and prove the Rational Root Theorem (it's OK if you give the detailed proof for just 1/2 of it).

(B) Show that if a, b are positive integers, then there exist integers m, n so that $ma + nb = \gcd(a, b)$.

(C) Show that if p is a prime number and $p|(ab)$ then $p|a$ or $p|b$. (You can use the result in (B) for proving (C).)