Name: ____________________________ **Panther ID:** ______________

Exam 2 MAT 3501 Fall 2017

1. (22 pts) Begin with a line segment and replace the middle third by an open square as shown (picture will be drawn on the board). Now repeat the process on each segment in the resulting figure.

(a) (4 pts) Draw the figure you get in the next iteration.

(b) (6 pts) If this process is continued indefinitely, what is the fractal dimension of the resulting fractal?

(c) (6 pts) Guess what you think the final fractal will look like.

(d) (6 pts) Find the "maximal height" of the fractal (that is, the maximal vertical distance from the initial line segment).

2. (22 pts) (a) (4 pts) Give the definition of an algebraic number.

(b) (6 pts) Show that $x = \sqrt[3]{2 + \sqrt{5}}$ is an algebraic number.

(c) (6 pts) State Gelfond's Theorem and explain how it is used to prove that $\sqrt{3}^{\sqrt{2}}$ is transcendental.

(d) (6 pts) Decide if the following statement is True or False. Then give a brief justification of your answer: "The product of any two transcendental numbers is transcendental. "

3. (14 pts) The point in a lunar orbit nearest the surface of the moon is called *perilune* and the point farthest to the surface is called *apolune*. The Apollo 11 spacecraft was placed in an elliptical lunar orbit with perilune altitude 110 km and apolune altitude 314 km (both altitudes measure the distance above the surface of the moon). Find an equation for this ellipse if the radius of the moon is 1728 km and the center of the moon is at one focus.

4. (22 pts) (a) (4 pts) Express $\frac{2-3i}{3+4i}$ in the $a+bi$ form.

(b) (6 pts) Find all complex solutions of $w^4 = -\sqrt{3} - i$. (Ok to leave your answer in polar or exponential form for this one).

(c) (6 pts) If $z_1 = 2 - 3i$ and $z_2 = 3 + 4i$, find z_3, so that z_1, z_2, z_3 are the vertices of an equilateral triangle.

(d) (6 pts) Recall that the principal logarithm of z, Logz, is defined by

$$\mathrm{Logz} = \ln \mathrm{r} + \mathrm{i}(\theta), \ \text{ if } \mathrm{z} = \mathrm{re}^{\mathrm{i}\theta},$$

and the principal value of z^w is defined by $z^w = e^{w\mathrm{Logz}}$. Find the principal value of i^i.

5. (24 pts) Choose TWO of the following three (12 pts each)

(A) State Euler formula and explain how one can justify this formula with series.

(B) Give either Archimedes' proof that the area of a disk is πr^2, or give a Calculus justification of the same fact (e.g. the one with regular n-gons inscribed in the disk and limit as $n \to \infty$.)

(C) A bank account pays an annual interest rate r percent, compounded continuously. If the principal is P, find, with proof, the balance $B(t)$ in the bank account after t years.

You can use without proof the limit $\lim_{x\to+\infty} \left(1+\dfrac{1}{x}\right)^x = e$