Name: _____ **Panther ID:** _____

Exam 2 MAT 3501 Fall 2018

1. (25 pts) For each of the following, answer if the statement is True or False. Then give a brief justification of your answer. (2 pts answer, 3 pts justification)

(a) $e^{i\pi} + 1 = 0$. **True** **False**

Justification:

(b) Any irrational number is transcendental. **True** **False**

Justification:

(c) The remainder of $x^{2018} + 1$ when divided by $x + 1$ is 2. **True** **False**

Justification:

(d) There is no polynomial $p(x)$ of degree 3 and with real coefficients, so that $p(1) = p(2) = p(3) = 0$ and $p(4) = 14$.

True **False**

Justification:

(e) $3^{\sqrt{3}}$ is transcendental. **True** **False**

Justification:

2. (12 pts) Factor and find all solutions (real or complex) of the equation $x^5 - 4x = 0$.

3. (16 pts) (a) (4 pts) Express $\frac{1-2i}{1+2i}$ in the $a+bi$ form.

(b) (6 pts) Find all complex solutions of $w^3 = i$ and picture them in the complex plane.

(d) (6 pts) Recall that the principal logarithm of z, $\mathrm{Log\,z}$, is defined by

$$\mathrm{Log\,z} = \ln \mathrm{r} + \mathrm{i}(\theta), \text{ if } \mathrm{z} = \mathrm{re}^{\mathrm{i}\theta},$$

and the principal value of z^w is defined by $z^w = e^{w\mathrm{Log\,z}}$. Find the principal value of $(-1)^i$.

4. (12 pts) Use the polar form to simplify

$$\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{2018}$$

For full credit, put your final answer in the form $a + bi$.

5. (24 pts) Consider the equation $x^5 + x + 5 = 0$ and let x_1, x_2, x_3, x_4, x_5 be its roots. (Don't try to find the roots explicitly, you will not be able to do it.)

(a) (6 pts) Find the sum $x_1 + x_2 + x_3 + x_4 + x_5$ and the product $x_1x_2x_3x_4x_5$.

(b) (6 pts) Show that at least one of the roots is real.

(c) (6 pts) Show that none of the roots is rational.

(d)* (6 pts) Show that **exactly** one of the roots is real.

6. (18 pts) Choose ONE of the following proofs. You can choose to do TWO proofs with the second score giving possible bonus towards a previous problem with a lower score.

(A) State and prove the Rational Root Theorem (it's OK if you give the detailed proof for just 1/2 of it).

(B) State and prove the quadratic formula.

(C) State the Fundamental Theorem of Algebra and prove the easy part.